MATH 120 2020-2 Quiz 4

Duration $\backsim$ 20 min.

- Write your NAME, SURNAME, ID and SECTION.
- Write the QUESTION completely on your solution paper.
- Upload your solutions to Gradescope as a SINGLE PAGE.

Question

Let $f(x, y) = xy + e^{2x+3y}$.

1. Approximate $f(0.1, -0.1)$
2. Determine the minimum rate of change of $f(x, y)$ at the point $(0, 0)$.

Answer

1. Linearization of $f(x, y)$ at (a, b) :

$$L(x, y) = f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b)$$

It is known that $f(x, y) \approx L(x, y)$, so in order to approximate $f(0.1, -0.1)$ we will use the linearization at $(0, 0)$. First, we should evaluate first order derivatives;

- $f_x(x, y) = y + 2e^{2x+3y}$ so $f_x(0, 0) = 0 + 2e^0 = 2$
- $f_y(x, y) = x + 3e^{2x+3y}$ so $f_y(0, 0) = 0 + 3e^0 = 3$

We have

$$L(x, y) = f(0, 0) + f_x(0, 0)x + f_y(0, 0)y = 1 + 2x + 3y$$

Therefore $f(0.1, -0.1) \approx L(0.1, -0.1) = 1 + 0.2 - 0.3 = 0.9$

2. Let f be differentiable function then for an unit vector u

$$D_u f = \nabla f \cdot u = |\nabla f||u|cos\theta = |\nabla f|cos\theta$$

where θ is the angle between ∇f and u. The minimum rate of change occurs when $\theta = \pi$.

Observe that $f_x(x, y) = y + 2e^{2x+3y}$ and $f_y(x, y) = x + 3e^{2x+3y}$ are continuous in a neighborhood of $(0, 0)$ which implies $f(x, y)$ is differentiable at $(0, 0)$.

Thus the minimum rate of change of f at $(0, 0)$ is

$$-|\nabla f(0, 0)| = -|\langle f_x(0, 0), f_y(0, 0)\rangle| = -|\langle 2, 3\rangle| = -\sqrt{4 + 9} = -\sqrt{13}$$