

Quiz #1 - Sample Solution

Question: Let $S_n = \frac{n}{2}$ be the sequence of partial sums for the series $\sum_{n=1}^{\infty} a_n$. Is the series $\sum_{n=1}^{\infty} \frac{a_n}{S_n}$ convergent or divergent.

Solution: Since $S_n = \frac{n}{2}$, then $S_{n-1} = \frac{n-1}{2}$ and we get

$$\boxed{a_1 + a_2 + \dots + a_{n-1} + a_n = \frac{n}{2}} \quad (1) \quad \&$$

$$\boxed{a_1 + a_2 + \dots + a_{n-1} = \frac{n-1}{2}} \quad (2)$$

Subtracting (2) from (1) gives

$$a_n = \frac{n}{2} - \frac{n-1}{2} \Rightarrow a_n = \frac{1}{2} \text{ for all } n \geq 1.$$

$$\text{Now the given series becomes } \sum_{n=1}^{\infty} \frac{a_n}{S_n} = \sum_{n=1}^{\infty} \frac{1/2}{n/2} = \sum_{n=1}^{\infty} \frac{1}{n},$$

which is a "harmonic series". Hence, it is divergent.

OR

$$\sum_{n=1}^{\infty} \frac{1}{n} \text{ diverges by "p-test" as } p=1.$$