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Help Room Hours: Mon. 11-40 – 12-30

Tues. 14.40 – 15-40.

REC – I

#1: Solve the following inequalities:

(a) $\dfrac{1}{2x+1} \geq 1 - x.$

$$\frac{1}{2x+1} + \frac{x-1}{1}_{(2x+1)} \geq 0 \Rightarrow \frac{1 + \overbrace{(x-1)(2x+1)}^{2x^2-x-1}}{2x+1} \geq 0.$$

$$\Rightarrow \frac{2x^2 - x}{2x+1} \geq 0. \qquad \frac{x(2x-1)}{2x+1} \geq 0.$$

$x = 0$ & $x = 1/2$. $x = -1/2$.

x	$-\infty$		$-1/2$		0		$1/2$		$+\infty$
$f(x)$		$-$		$+$		$-$		$+$	

Solution set: $(-1/2, 0] \cup [1/2, \infty)$.

(b) $|x+3| - 2 > 3x$

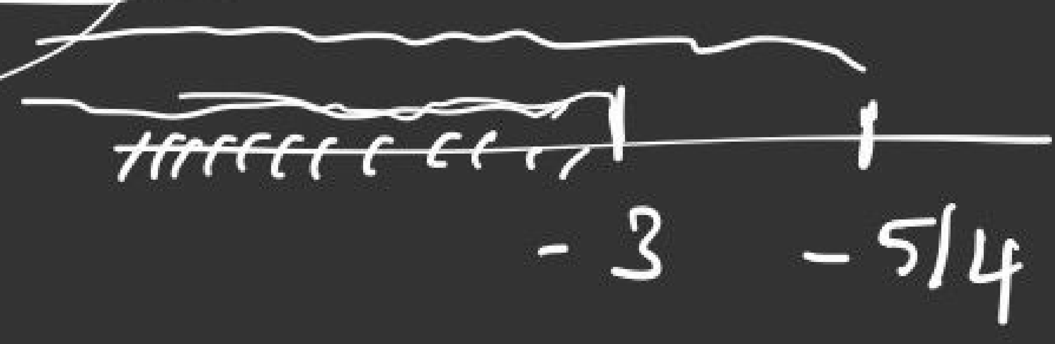
$|x+3| > 3x+2$.

when $x > -3$; $x+3 > 3x+2 \Rightarrow 1 > 2x$

$\Rightarrow 1/2 > x$ $1/2 > x > -3$. $(-3, 1/2)$.

when $x < -3$; $-x-3 > 3x+2 \Rightarrow -5 > 4x$.

$\Rightarrow -5/4 > x$



$x < -3$

$(-\infty, -3)$.

when $x = -3$;

$|-3+3| > 3\cdot(-3)$

$0 > -9$ ✓.

$\{-3\}$.

Solution set: $(-\infty, -3) \cup \{-3\} \cup (-3, 1/2)$

$= (-\infty, 1/2)$.

#c: $\dfrac{x^3 - x^2 + 4}{x+3} \le 1.$

$x^3 - x^2 + 4 \le x + 3.$

$x^3 - x^2 - x + 1 \le 0.$

$x^2(x-1) - (x-1) \le 0.$

wrong way!

$(x-1)^2(x+1) \le 0.$

$x = 1$ $x = -1$.

$$\frac{x^3 - x^2 + 4}{x+3} - 1 \le 0.$$

$$\frac{x^3 - x^2 + 4 - x - 3}{x+3} \le 0.$$

$$\frac{x^3 - x^2 - x + 1}{x+3} \le 0$$

$x^2(x-1) - (x-1)$

$(x-1)(x^2-1)$

$(x-1)(x+1)$

$$\Rightarrow \frac{(x-1)^2(x+1)}{x+3} \le 0.$$

$x = 1 \rightarrow$ double root.

$x = -1$

$x = -3$

x	$-\infty$		-3		-1		1		$+\infty$
$f(x)$		$+$	0	$-$	0	$+$	0	$+$	

usually forgotten!

Solution set: $[-3, -1] \cup \{1\}.$

#2: Write an equation for the line through the points

$$\overset{x_1\ y_1}{(-1, 5)} \ \& \ \overset{x_2\ y_2}{(0, 3)} \qquad (x_1, y_1) \ \& \ (x_2, y_2)$$

$$\underset{\text{slope}}{\underset{\downarrow}{y = mx + b}} \qquad m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-5}{0-(-1)} = -2.$$

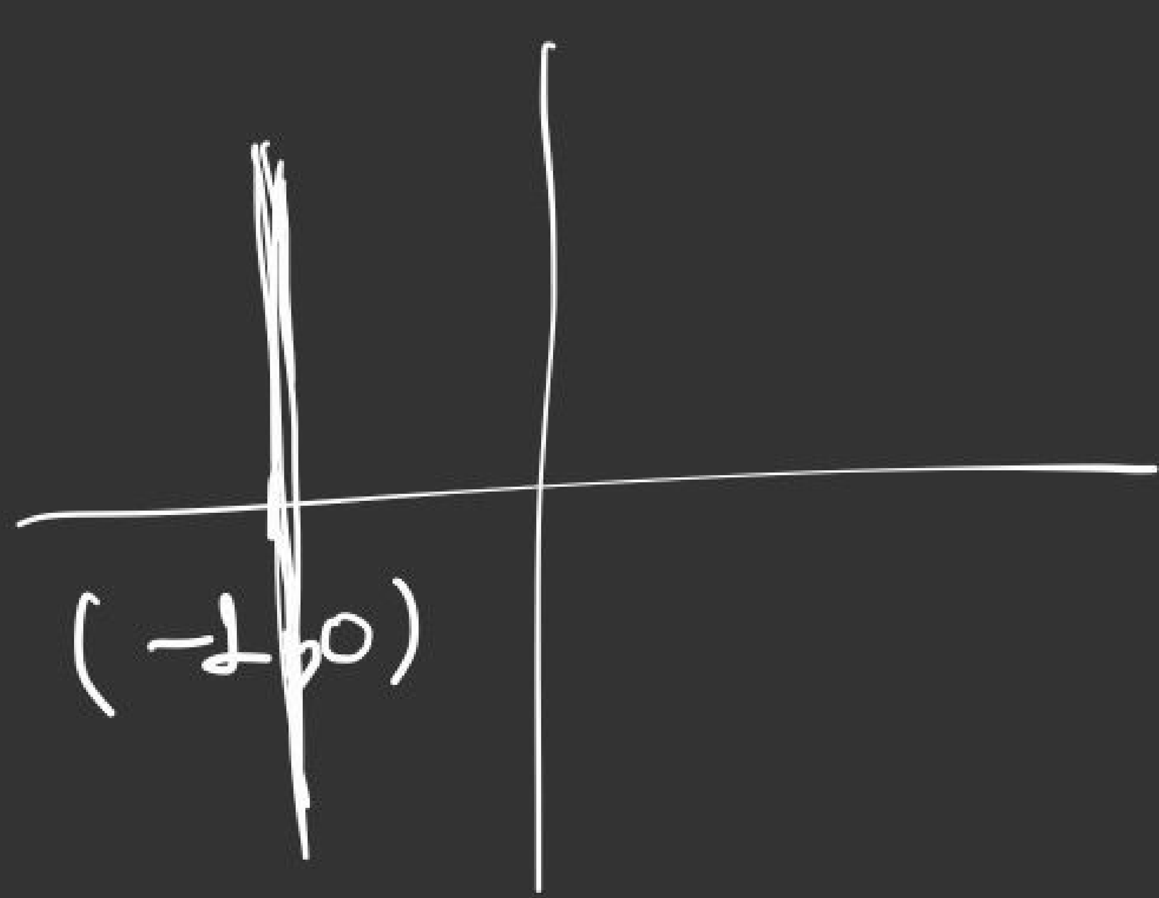
$$y = -2x + b \xrightarrow{(0,3)} 3 = -2 \cdot 0 + b.$$

$$b = 3.$$

$$\boxed{y = -2x + 3}$$

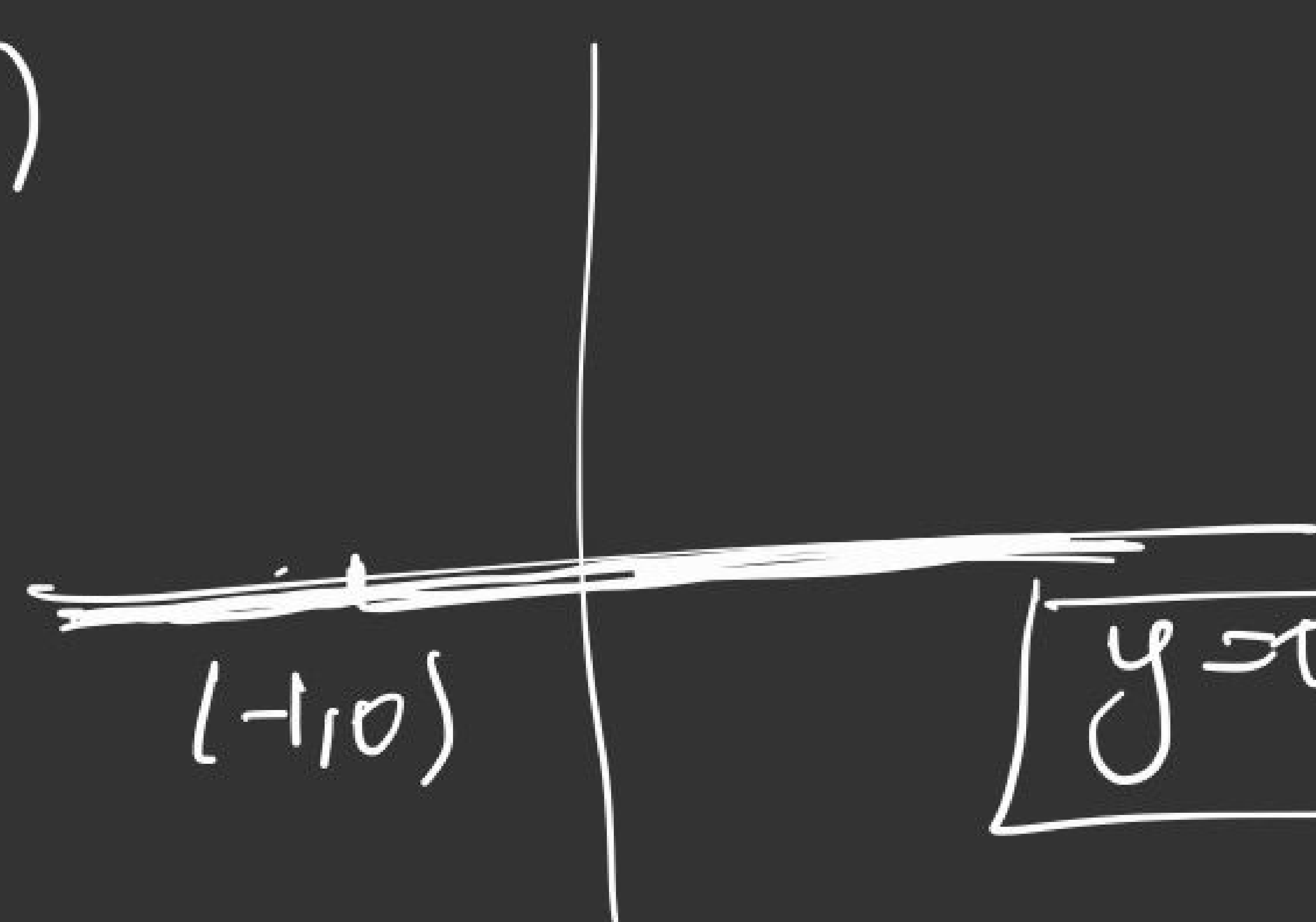
#3: Find the equation for (a) the vertical line and (b) the horizontal line through the point $(-1, 0)$.

(a)



$$\boxed{x = -1.}$$

(b)



$$\boxed{y = 0}$$

#4: Find the equation for the line through $P(-1,3)$ that is perpendicular to the line $y+x+2=0$. Find the x & y-intercepts of this line.

$y = \underbrace{-}_{m_1}x - 2.$ $\quad \boxed{m_1 = -1}$.

Let m_2 be the slope of the line passing through $(-1,3)$ & perpendicular to $y=-x-2$.

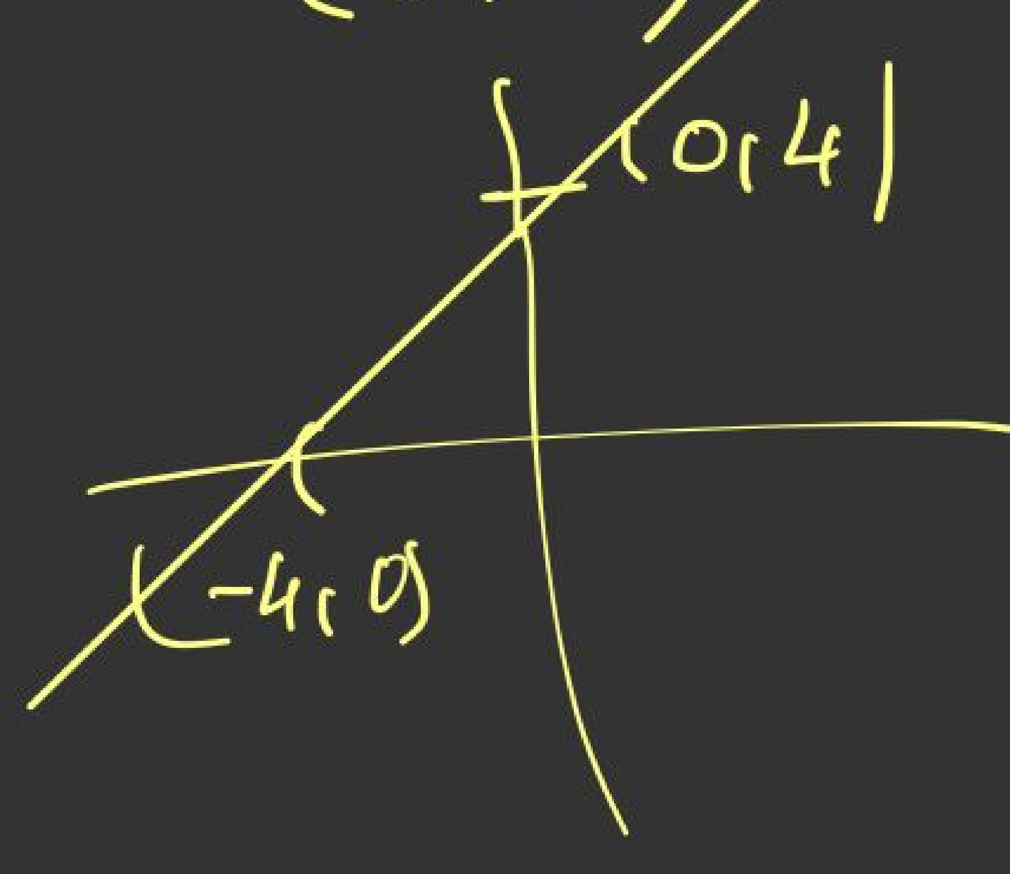
$\boxed{\underbrace{m_1}_{-1} \cdot m_2 = -1.}$ $\Rightarrow$ $\boxed{m_2 = 1}$.

$y = \underbrace{m_2}_{1} x + b$ $\quad \Rightarrow \quad y = x+b \xrightarrow{(-2,3)} 3 = -1 + b.$

$\Rightarrow \boxed{b=4}$ $\qquad \boxed{y = x+4}$.

For x intercept: $\quad y=0 \Rightarrow x=-4. \quad (-4,0)$

For y-intercept: $\quad x=0 \Rightarrow y=4. \quad (0,4)$



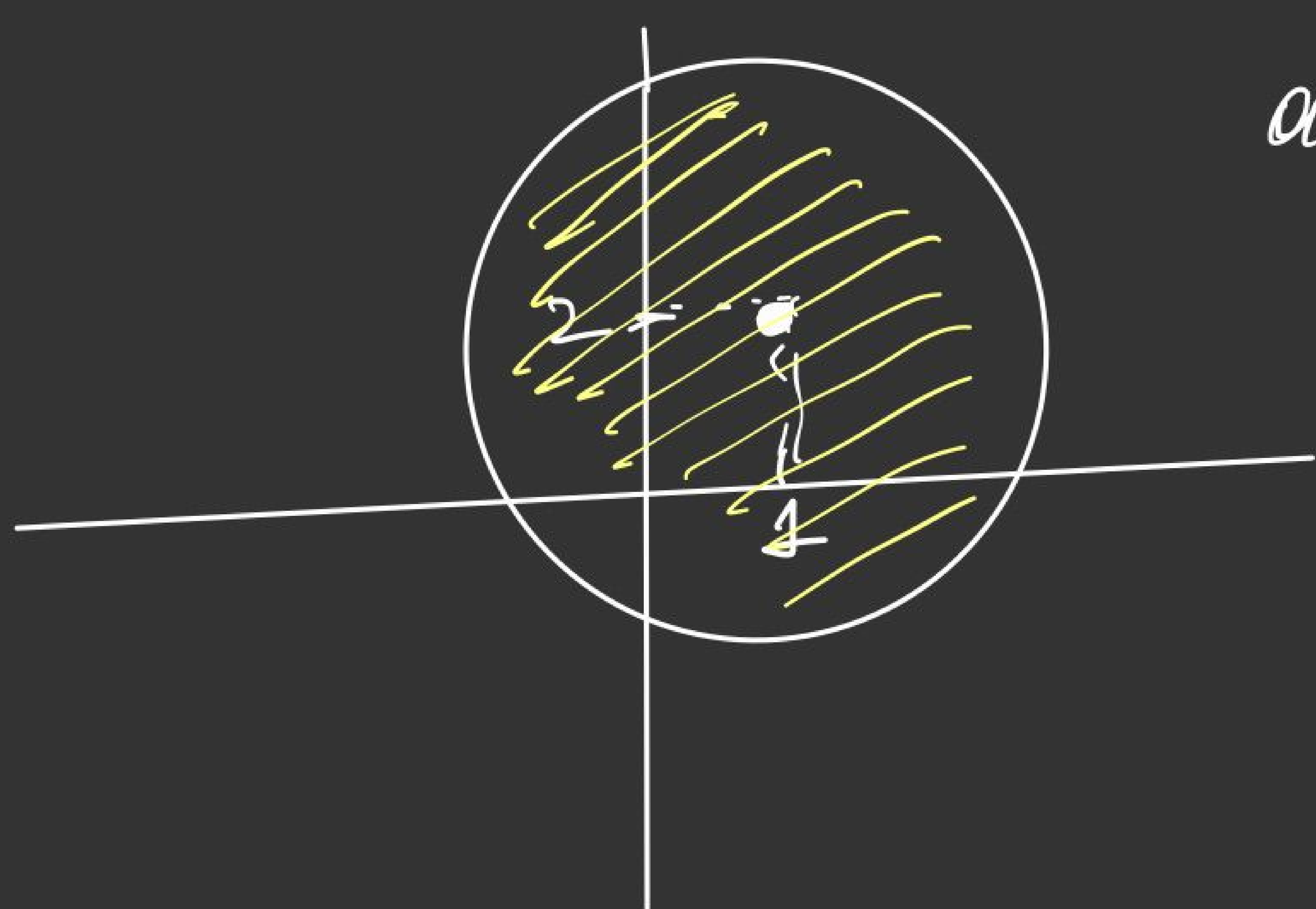
#5: Describe and sketch the regions defined by the followings:

(a) $x^2+y^2-2x-4y \le 4$.

$$x^2-2x+1+y^2-4y+4 \le 4+5.$$

$$(x-1)^2+(y-2)^2 \le 9 = 3^2.$$

$(x-1)^2+(y-2)^2 = 3^2 \to$ the equation of the circle centered at $(1,2)$ and with radius 3.



(b) $x^2+y^2 \le 4$ & $x^2+y^2 > 2y$.

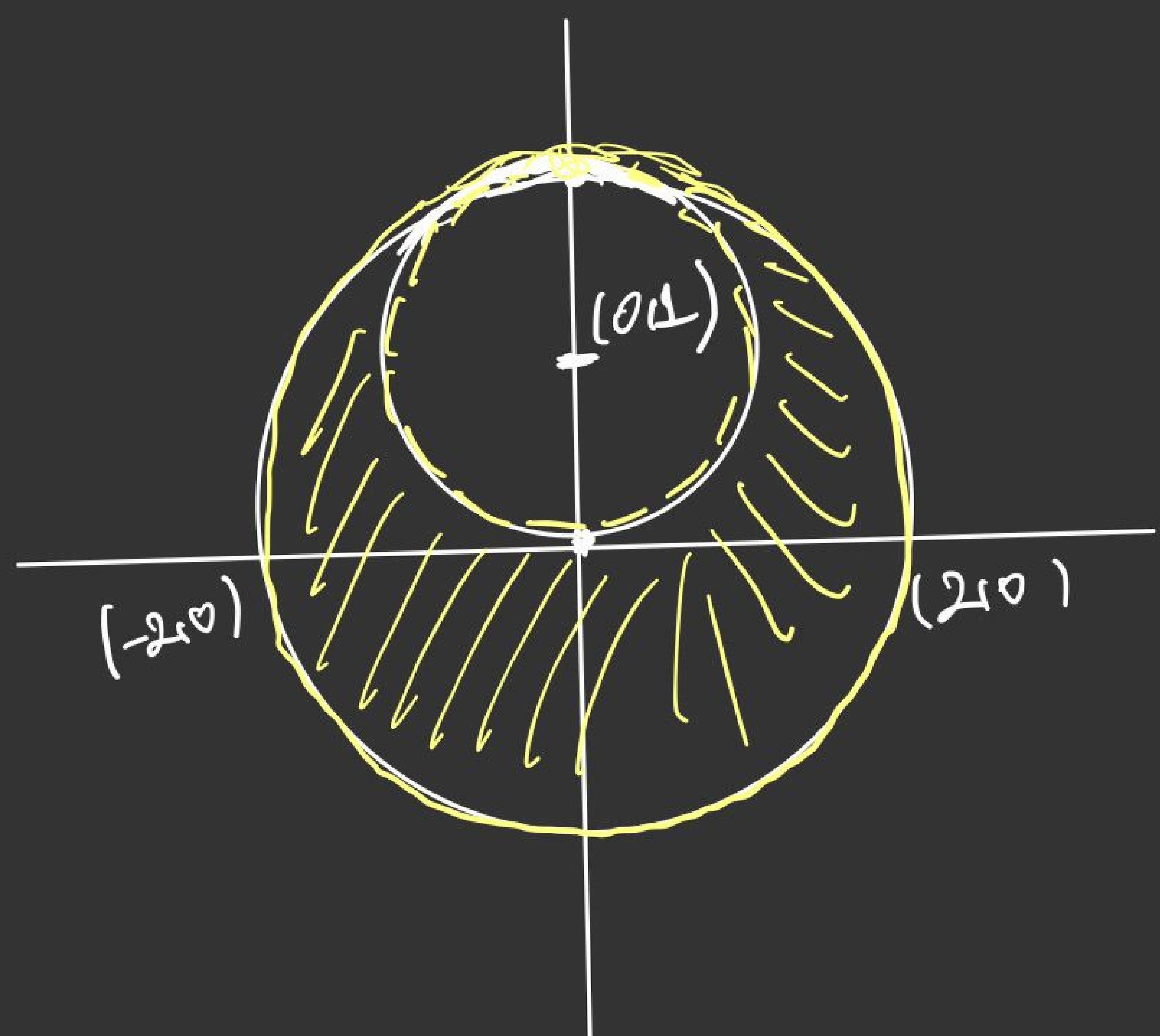
circle centered at origin with radius 2.

$x^2+y^2-2y \ge 0$.

$x^2+y^2-2y+1 > 1$.

$x^2+(y-1)^2 > 1$.

circle centered at $(0,1)$ with radius 1



(c) $x^2+y^2 > 2y$ & $y > 1+x$.

$x^2+y^2-2y+1 > 1$

$x^2+(y-1)^2 > 1$.

↓

<u>outside</u> the circle

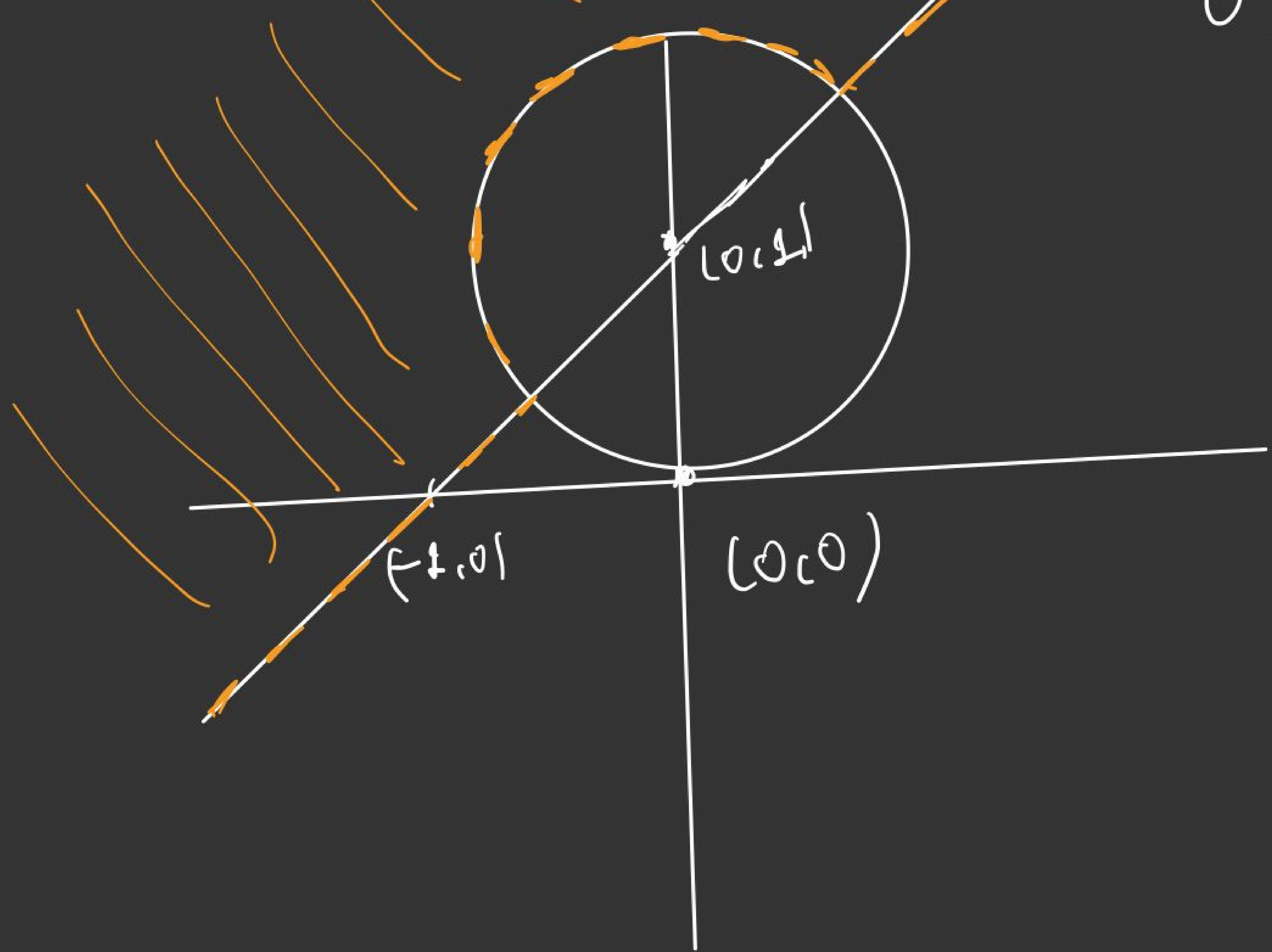
centered at $(0,1)$

with $r=1$.

$y = 1+x$.

$(0,1)$.

$(-1,0)$



$0 > 1+0$.

No!

#6: Find the points of intersection of pairs of curves

(a) $y = x^2 + 3$ & $y = 3x + 1$.

Equate the equations to get a common solution.

$x^2 + 3 = 3x + 1$. $\Rightarrow$ $x^2 - 3x + 2 = 0$.

$x \quad -2$

$x \quad -1$.

$(x-2)(x-1) = 0$. $x = 2$ & $x = 1$.

For $x = 2$, $y = 7$.

For $x = 1$, $y = 4$.

$\{(1,4)$ & $(2,7)\}$

$\downarrow$

points of intersection.

(b) $2x^2 + 2y^2 = 5$ & $xy = 1$.

$\boxed{y = 1/x \quad x \neq 0}$ $2x^2 + 2(1/x)^2 = 5$.

$2x^2 + \dfrac{2}{x^2} = 5 \Rightarrow \dfrac{2x^4 + 2}{x^2} = 5$.

$\downarrow$

$(x \neq 0)$.

$\Rightarrow 2x^4 + 2 = 5x^2 \Rightarrow 2x^4 - 5x^2 + 2 = 0$.

$2x^2 \quad -1$

$x^2 \quad -2$

$(2x^2-1).(x^2-2)=0.$

$x^2=1/2 \quad \& \quad x^2=2.$

$\Rightarrow \quad \underline{\underline{x=\mp 1/\sqrt{2}}} \quad \& \quad \underline{\underline{x=\mp\sqrt{2}}}.$

For $x=\mp 1/\sqrt{2} \Rightarrow y=\mp\sqrt{2}.$

For $x=\mp\sqrt{2} \Rightarrow y=\mp 1/\sqrt{2}.$

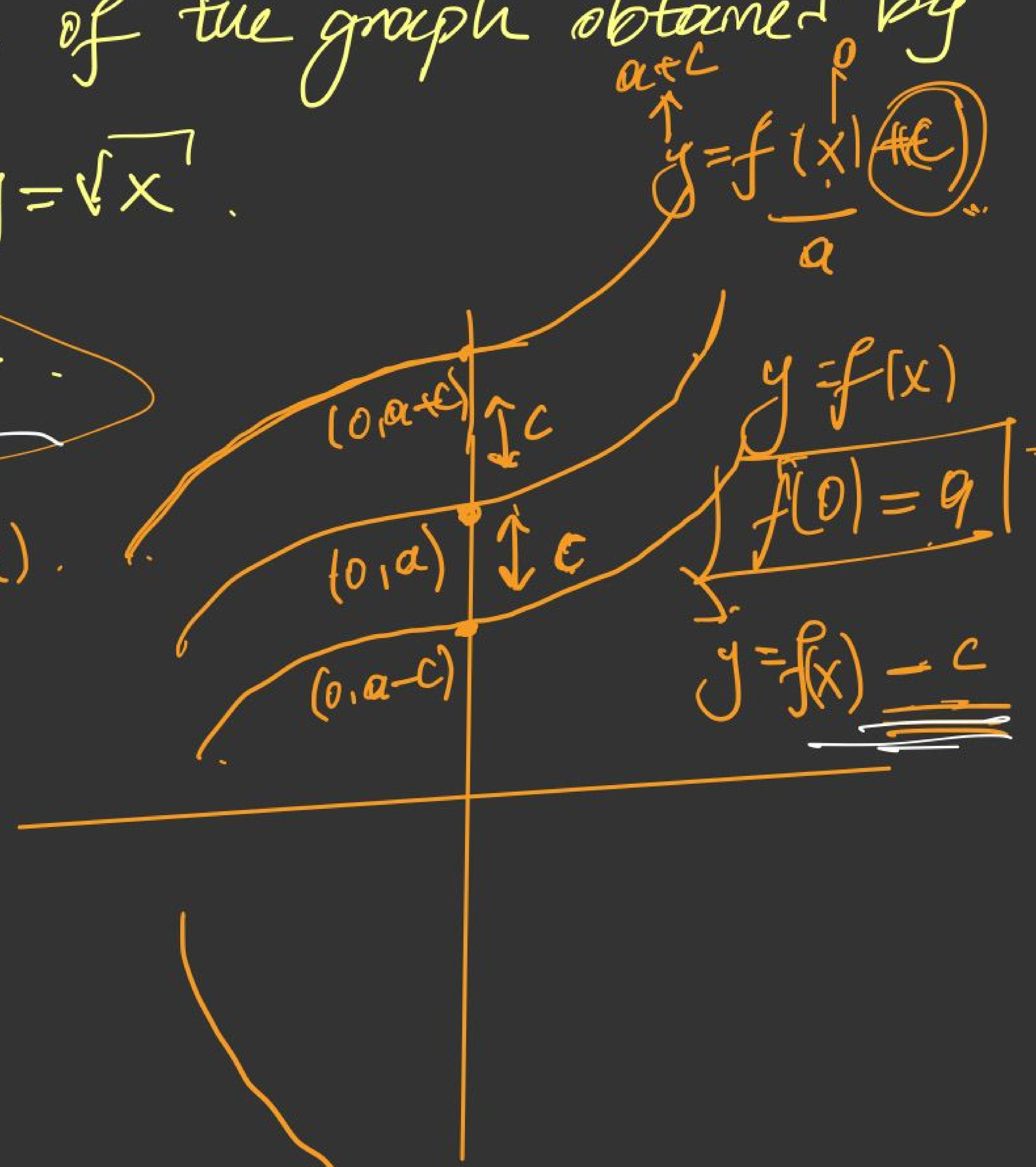
$\left\{ (1/\sqrt{2}, \sqrt{2}), (-1/\sqrt{2}, -\sqrt{2}), (\sqrt{2}, 1/\sqrt{2}), (-\sqrt{2}, -1/\sqrt{2}) \right\}$

points of intersection.

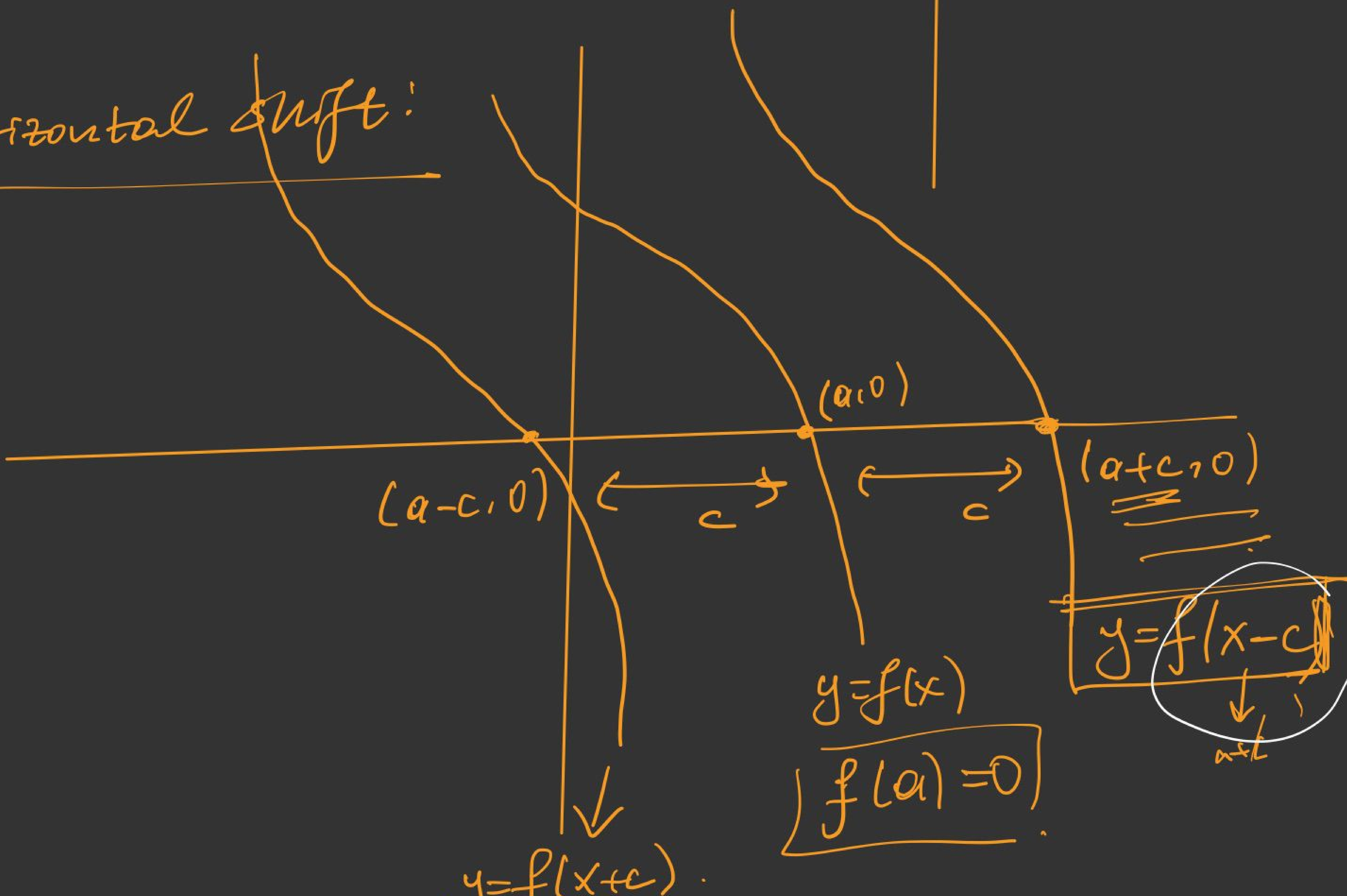
#7: Write an equation of the graph obtained by shifting the graph of $y=\sqrt{x}$.

(a) down 1, right 1.

Vertical shift: $y=f(x)$.



Horizontal shift:



$$\boxed{y=\sqrt{x-1}-1}$$

(b) down 2, left 4

$$y=\sqrt{x+4}-2.$$

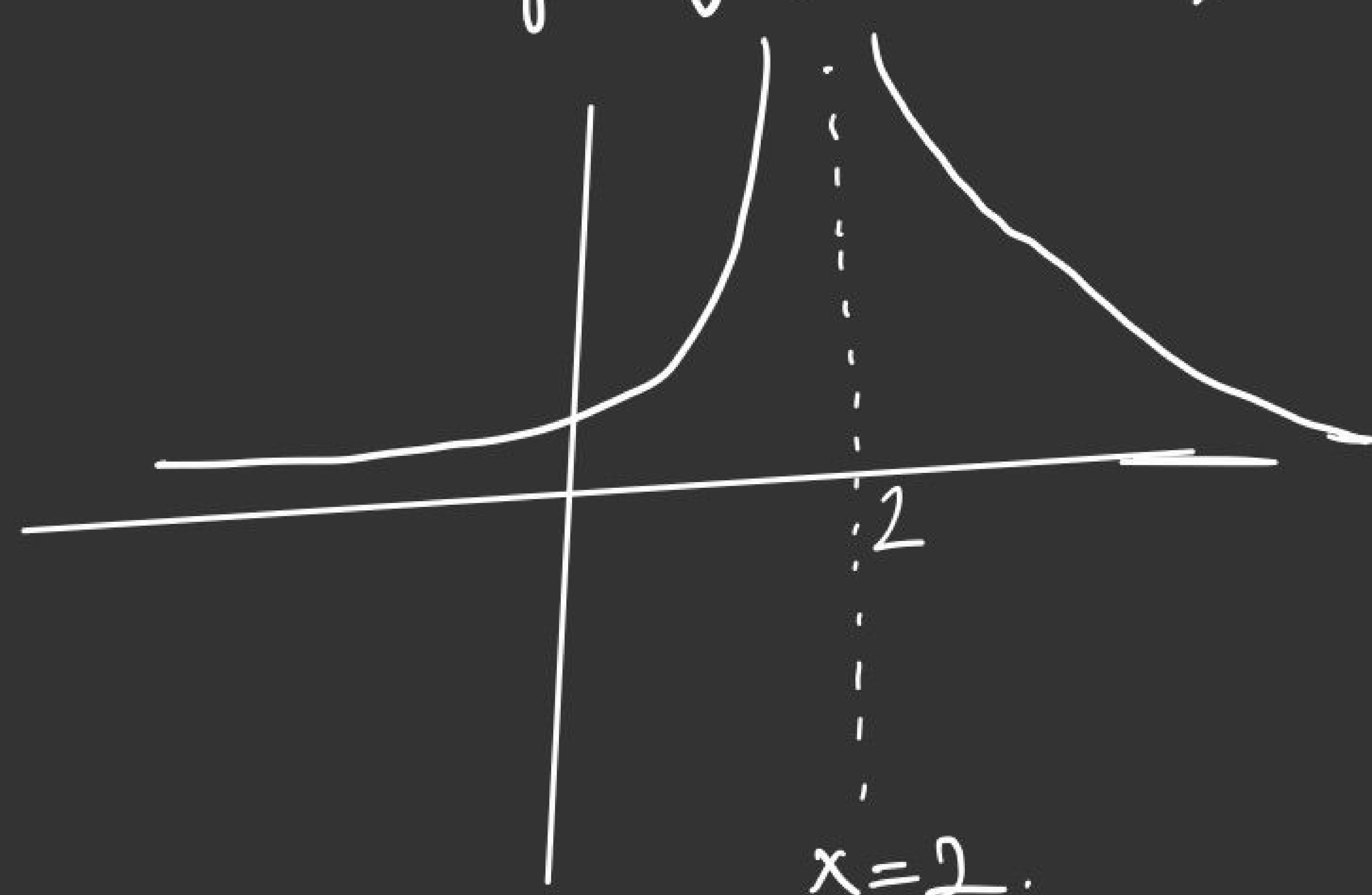
#8: Find the domain and the range of each function and sketch their graphs.

(a) $\dfrac{1}{|2-x|}=f(x)$.

Domain: $\text{Dom} f=\mathbb{R}-\{2\}$ since the absolute value function is defined on everywhere except at $x=2$.

Range: $|2-x|\geq 0$ for all $(\forall)$ $x\in \text{Dom} f$ therefore,

$\dfrac{1}{|2-x|}>0$ $\forall x\in \text{Dom} f$. and $\dfrac{1}{|2-x|}\neq 0$ for any $x\in\text{Dom} f$, $0\notin \text{Range}(f)$ therefore $\text{Range}(f)=(0,\infty)$.



(b) $y = 1 + \sin\left(x + \frac{\pi}{4}\right) = f(x)$.

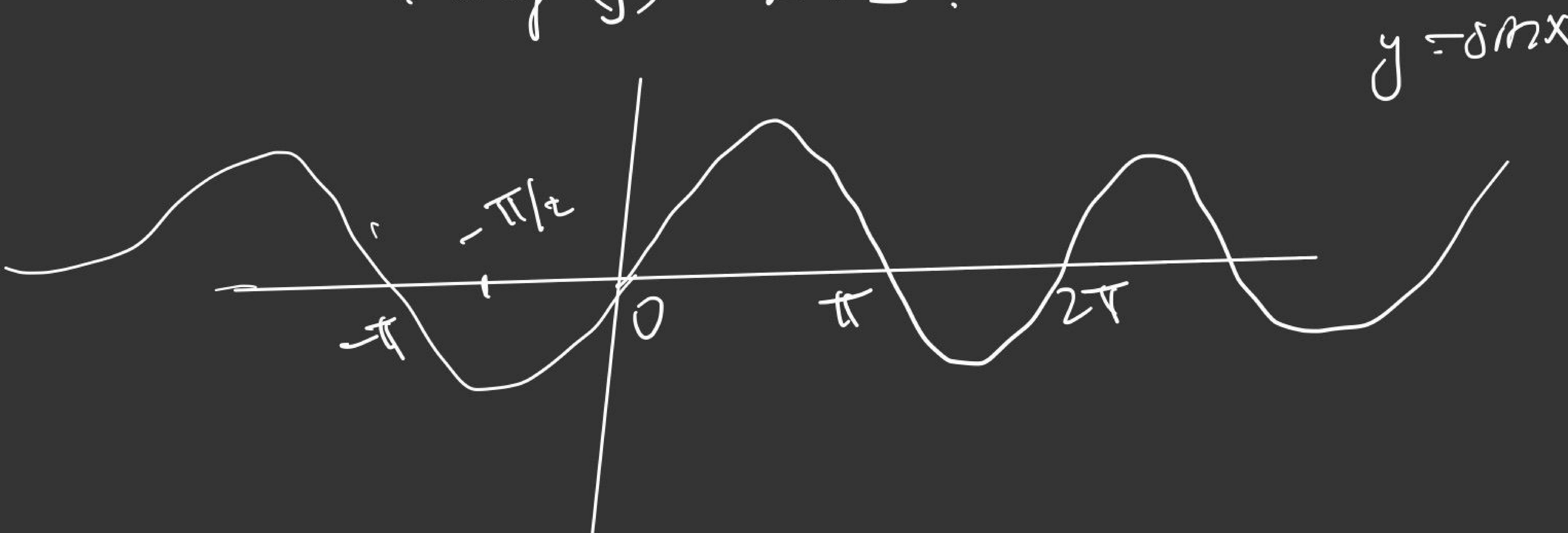
Domain: sine funct. is defined everywhere on $\mathbb{R}$ therefore, ~~f(x)~~. $\text{Dom } f = \mathbb{R}$.

Range: $-1 \le \sin x \le 1 \quad \forall x \in \cancel{\text{Dom}}\ \mathbb{R}$.

$$-1 \le \sin\left(x + \frac{\pi}{4}\right) \le 1.$$

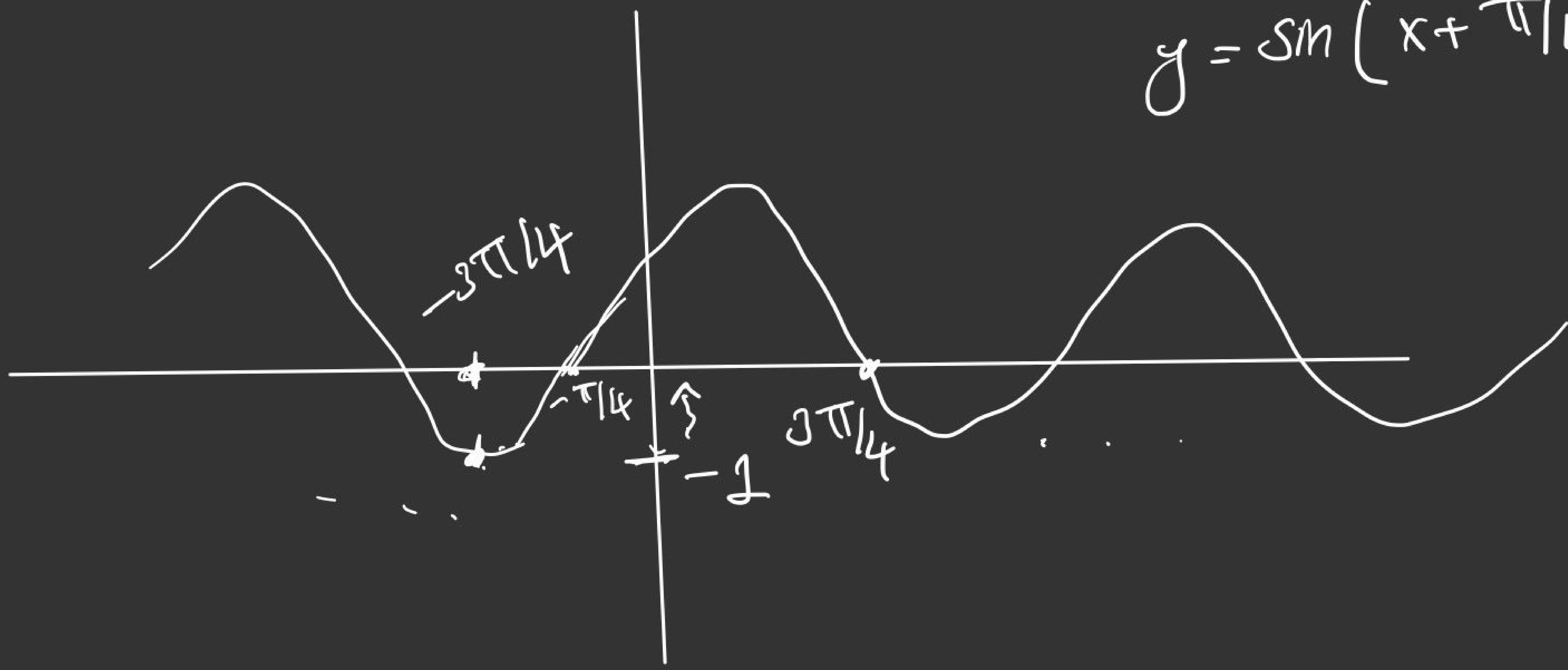
$$0 \le \underbrace{1 + \sin\left(x + \frac{\pi}{4}\right)}_{f(x)} \le 2.$$

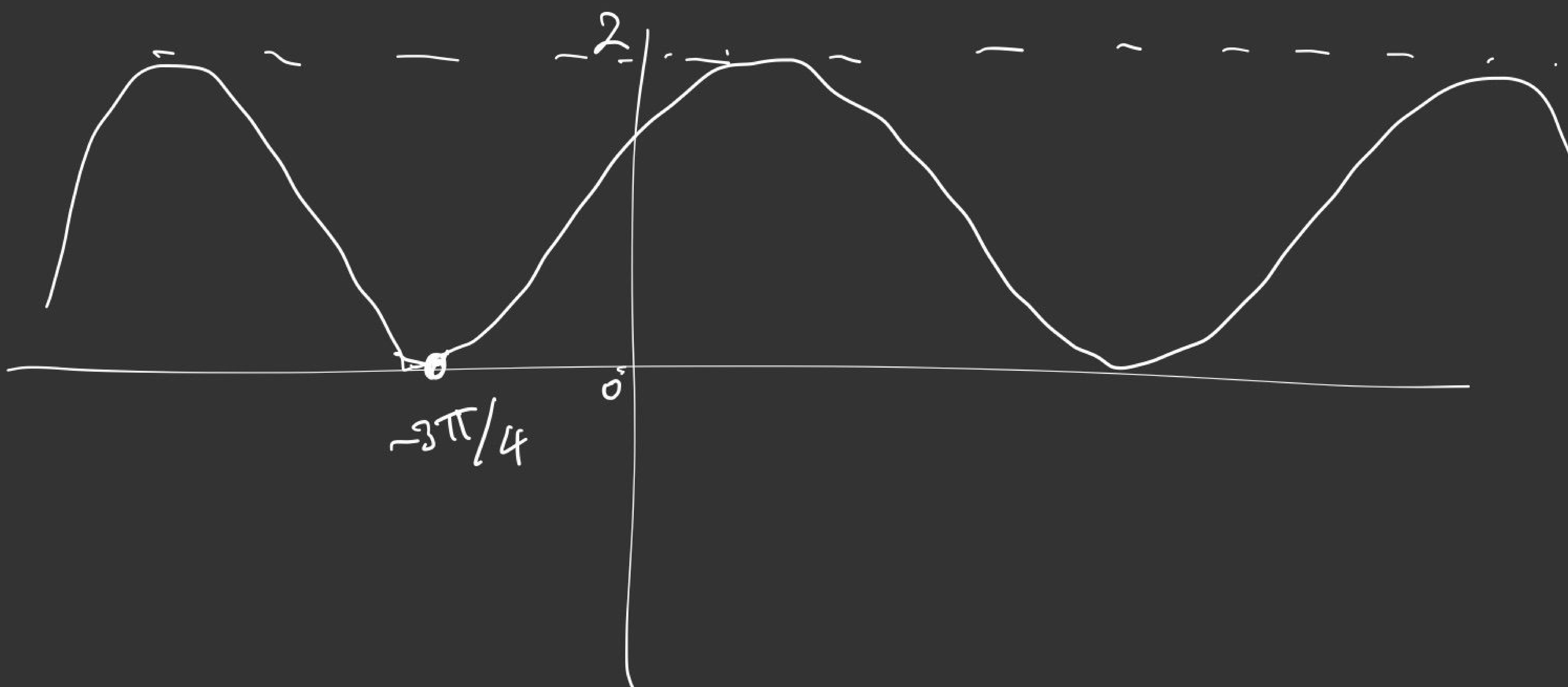
$$\text{Range}(f) = [0, 2].$$



shift left by $\pi/4$.

$y = \sin\left(x + \pi/4\right)$.





#9: Find $f \circ g$ and its domain where

$f(x) = x + \frac{1}{x}$ & $g(x) = \frac{x-1}{x+3}$.

$$(f \circ g)(x) = f(g(x)) = g(x) + \frac{1}{g(x)}$$

$$= \frac{x-1}{x+3} + \frac{x+3}{x-1} = \frac{2x^2+4x+10}{(x-1)(x+3)}.$$

$(x-1) \quad (x+3)$

$$\mathrm{Dom}(f \circ g) = \mathbb{R} \setminus \{1, -3\}.$$