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Help Room Hours: Mon. 11.40–12.30

Tues. 14.40–15.30.

REC–I

#1: Solve the following inequalities:

(a) $\dfrac{1}{2x+1} \geq 1-x.$

$$\frac{1}{2x+1} + \frac{x-1}{\underset{(2x+1)}{1}} \geq 0 \Rightarrow \frac{1}{2x+1} + \frac{\overbrace{(x-1)\cdot(2x+1)}^{2x^2-x-1}}{2x+1} \geq 0.$$

$$\frac{2x^2-x}{2x+1} \geq 0. \Rightarrow \frac{x(2x-1)}{2x+1} \geq 0.$$

$x=0.$

$x=1/2.$

$x=-1/2.$

x	$-\infty$		$-1/2$		0		$1/2$		$+\infty$
$f(x)$		$-$		$+$		$-$		$+$	

Solution set: $(-1/2, 0] \cup [1/2, \infty)$

(b) $|x+3| - 2 > 3x$

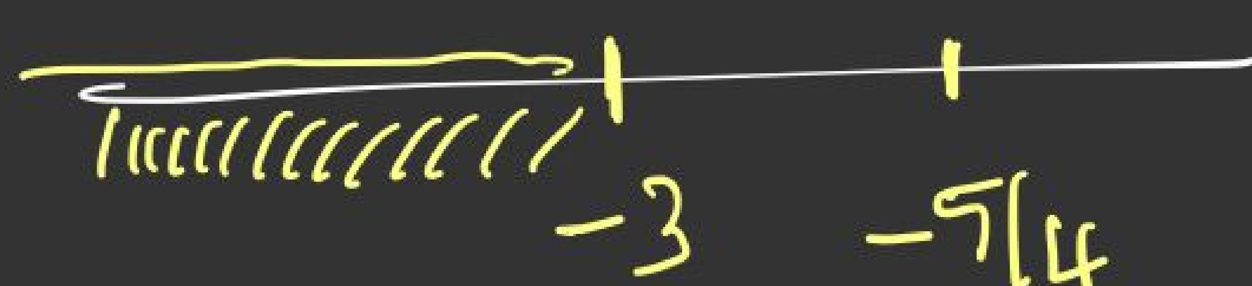
$|x+3| > 3x+2.$

when $x > -3$: $\quad x+3 > 3x+2 \Rightarrow 1 > 2x$

$\Rightarrow 1/2 > x \qquad -3 < x < 1/2 \qquad (-3, 1/2).$

when $x < -3$: $\quad -x-3 > 3x+2 \Rightarrow -5 > 4x$

$\Rightarrow -5/4 > x.$



$x < -3.$

$(-\infty, -3).$

when $x = -3$: $\quad |-3+3| > 3\cdot(-3)+2$

$0 > -7$ ✓.

$\{-3\}.$

Solution set: $(-\infty, -3) \cup \{-3\} \cup (-3, 1/2)$

$$\boxed{= (-\infty, 1/2).}$$

(c) $\dfrac{x^3-x^2+4}{x+3} \le 1.$

$$\frac{x^3-x^2+4}{x+3} - \underset{(x+3)}{\frac{1}{1}} \le 0 \Rightarrow \frac{x^3-x^2+4-x-3}{x+3} \le 0.$$

$$\Rightarrow \frac{x^3-x^2-x+1}{x+3} \le 0 \Rightarrow \frac{x^2(x-1)-(x-1)}{x+3} \le 0.$$

$$\frac{(x-1)\overbrace{(x^2-1)}^{(x-1)(x+1)}}{x+3} \le 0. \Rightarrow \frac{(x-1)^2(x+1)}{x+3} \le 0.$$

$x=1$ (double root)

$x=-1$

$x=-3$.

x	$-\infty$		-3		-1		1		$+\infty$.
$f(x)$		$+$	‖	$-$	0	$+$	0	$+$	

Solution set: $[-3,-1] \cup \{1\}$.

#2: write an equation for the line through the points $\overset{x_1}{(-1},\overset{y_1}{5)}$ & $\overset{x_2}{(0},\overset{y_2}{3)}$.

(x_1, y_1) & (x_2, y_2)

$$y = \underset{\text{slope}}{\underset{\downarrow}{m}}x + b$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

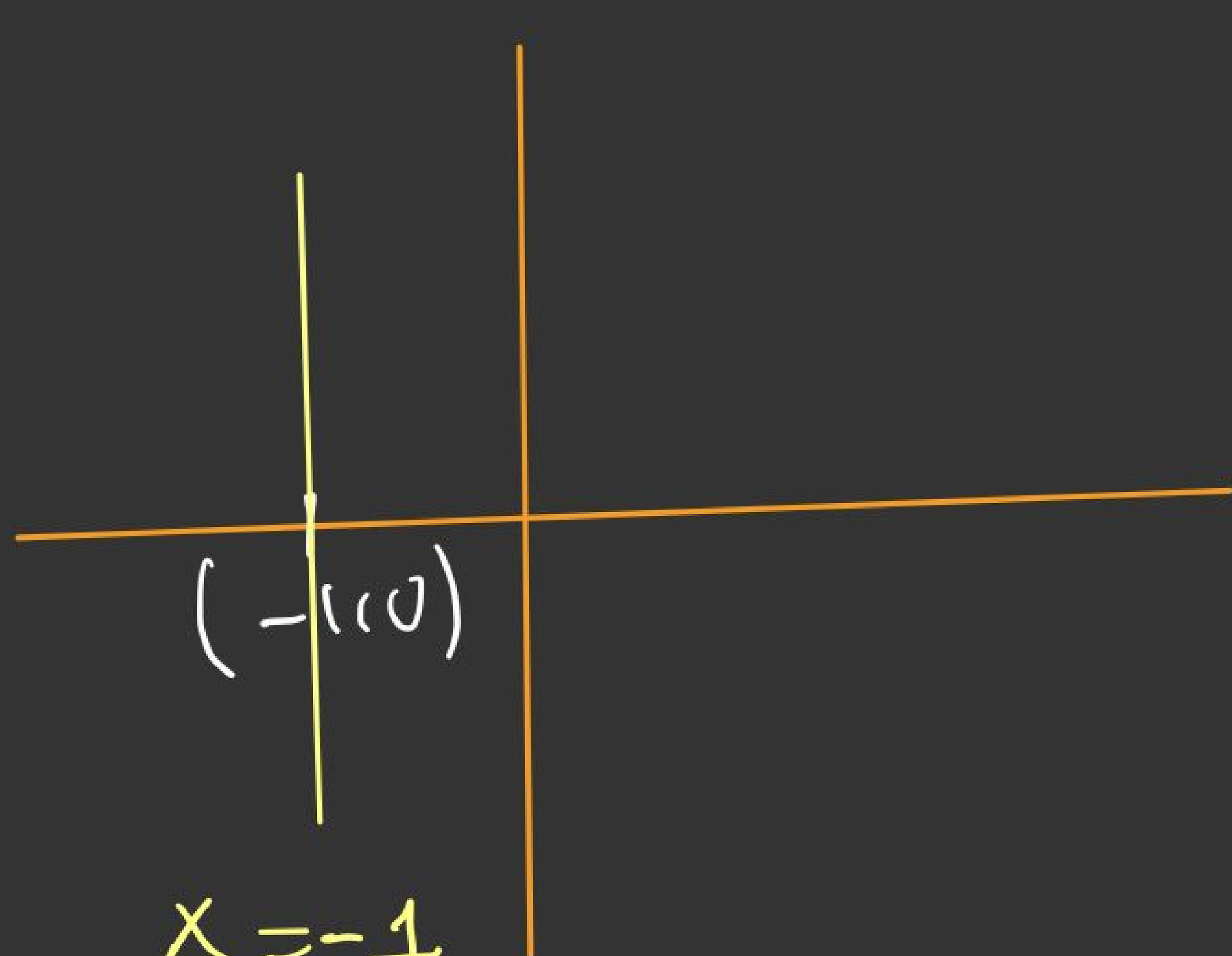
$$m = \frac{3-5}{0-(-1)} = -2.$$

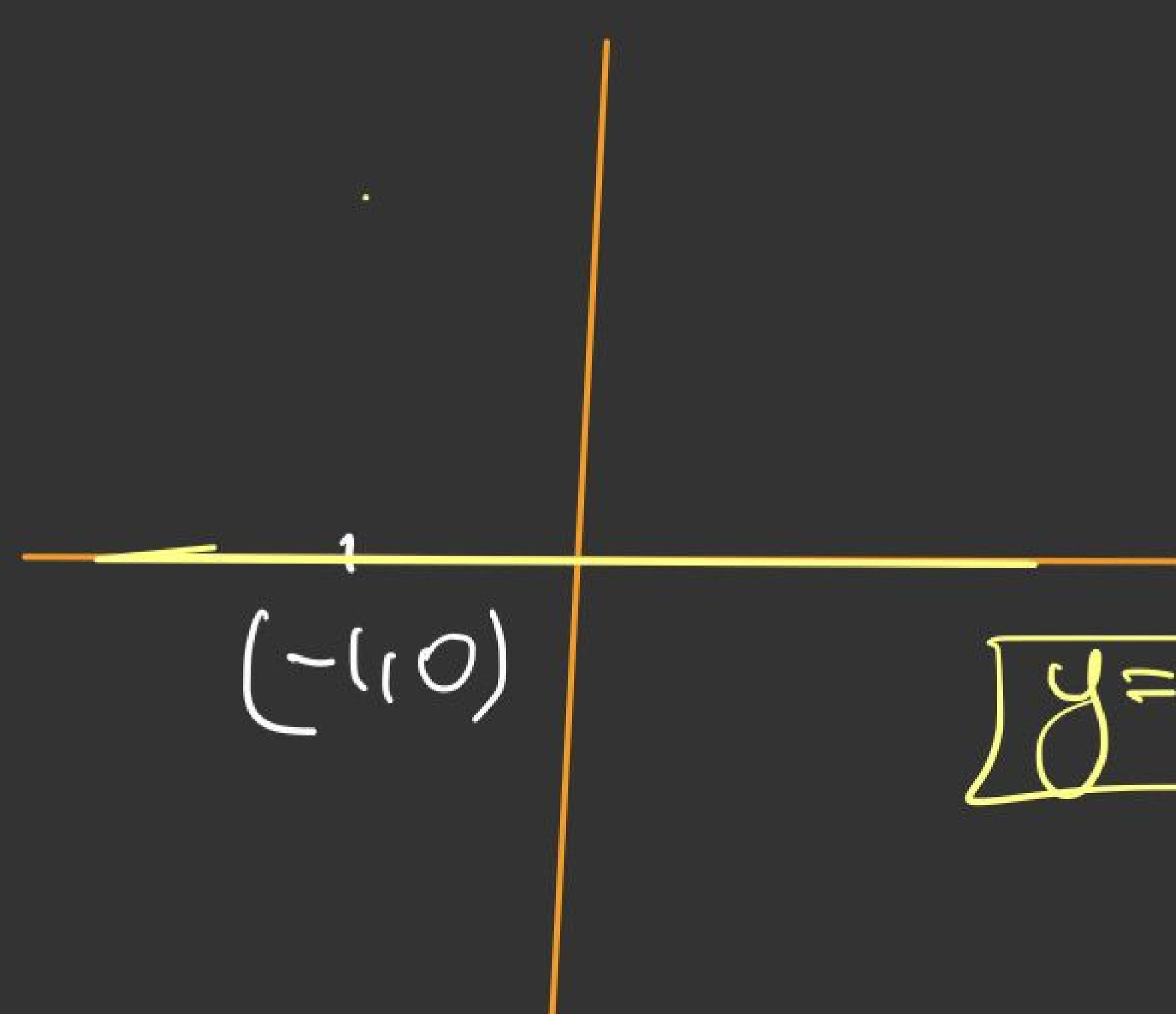
$$y = -2x + b \xrightarrow{(0,3)} 3 = -2\cdot 0 + b$$

$$\boxed{b = 3}$$

$$\boxed{y = -2x + 3}$$

#3: Find the equation for the vertical line and the horizontal line through the point $(-1,0)$





$\boxed{y=0}$

#4: Find the equation for the line through $P(-1,3)$ that is perpendicular to the line $y+x+2=0$. Find the x & y-intercepts of this line.

$$y = \underbrace{-x}_{m_1} - 2 . \quad m_1 = -1 .$$

Let m_2 be the slope of the line that is perp. to $y = -x-2$. Then we must have $\underbrace{m_1}_{-1} \cdot m_2 = -1$.

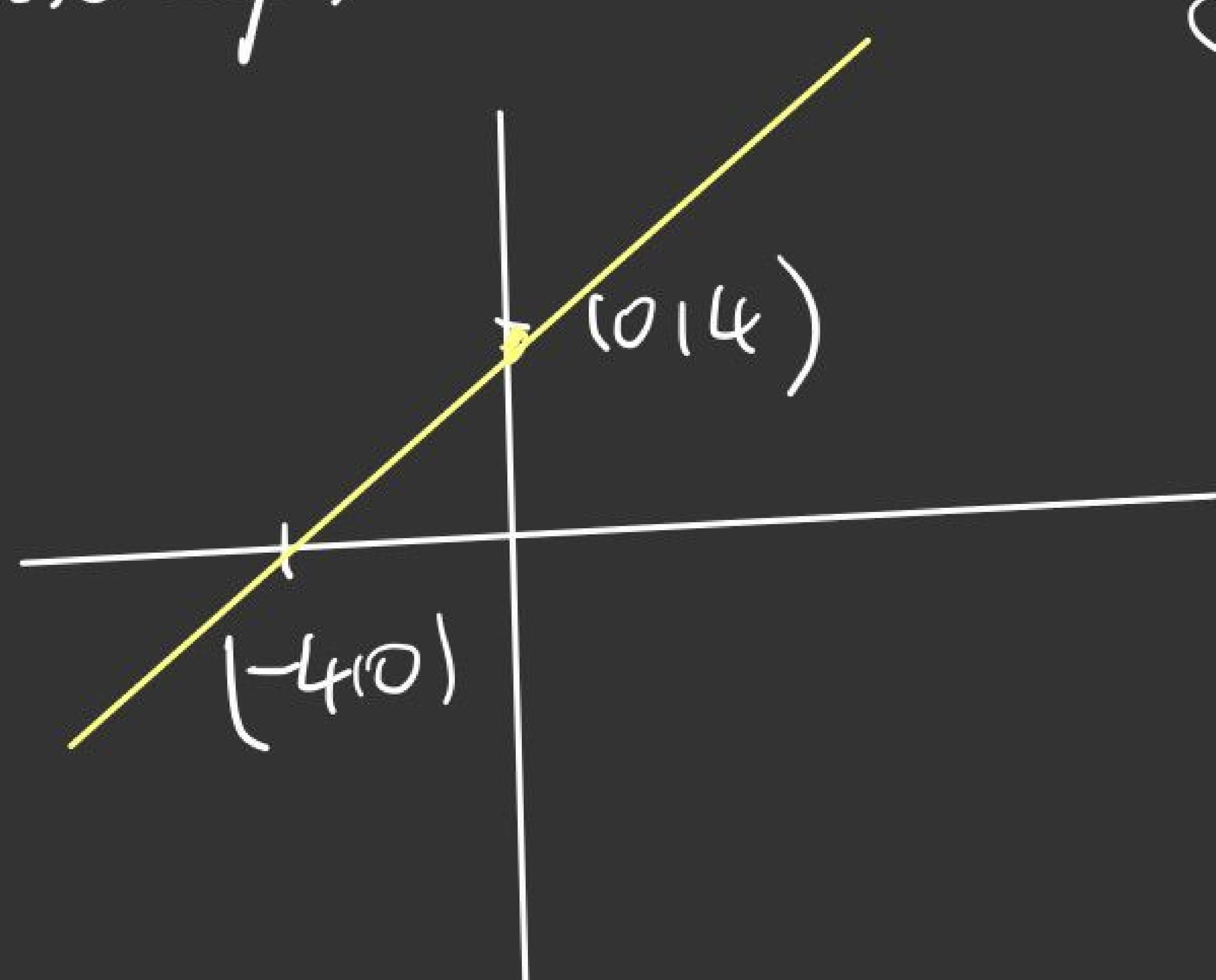
$$\Rightarrow \boxed{m_2 = 1} .$$

$$y = m_2 \cdot x + b \Rightarrow y = x + b \xrightarrow{(-1,3)} 3 = -1 + b$$

$$\Rightarrow \boxed{b = 4} \qquad \Rightarrow \boxed{y = x + 4} .$$

For x-intercept: $y = 0 \Rightarrow x = -4$ $\quad (-4,0)$

For y-intercept: $x = 0 \Rightarrow y = 4$ $\quad (0,4)$



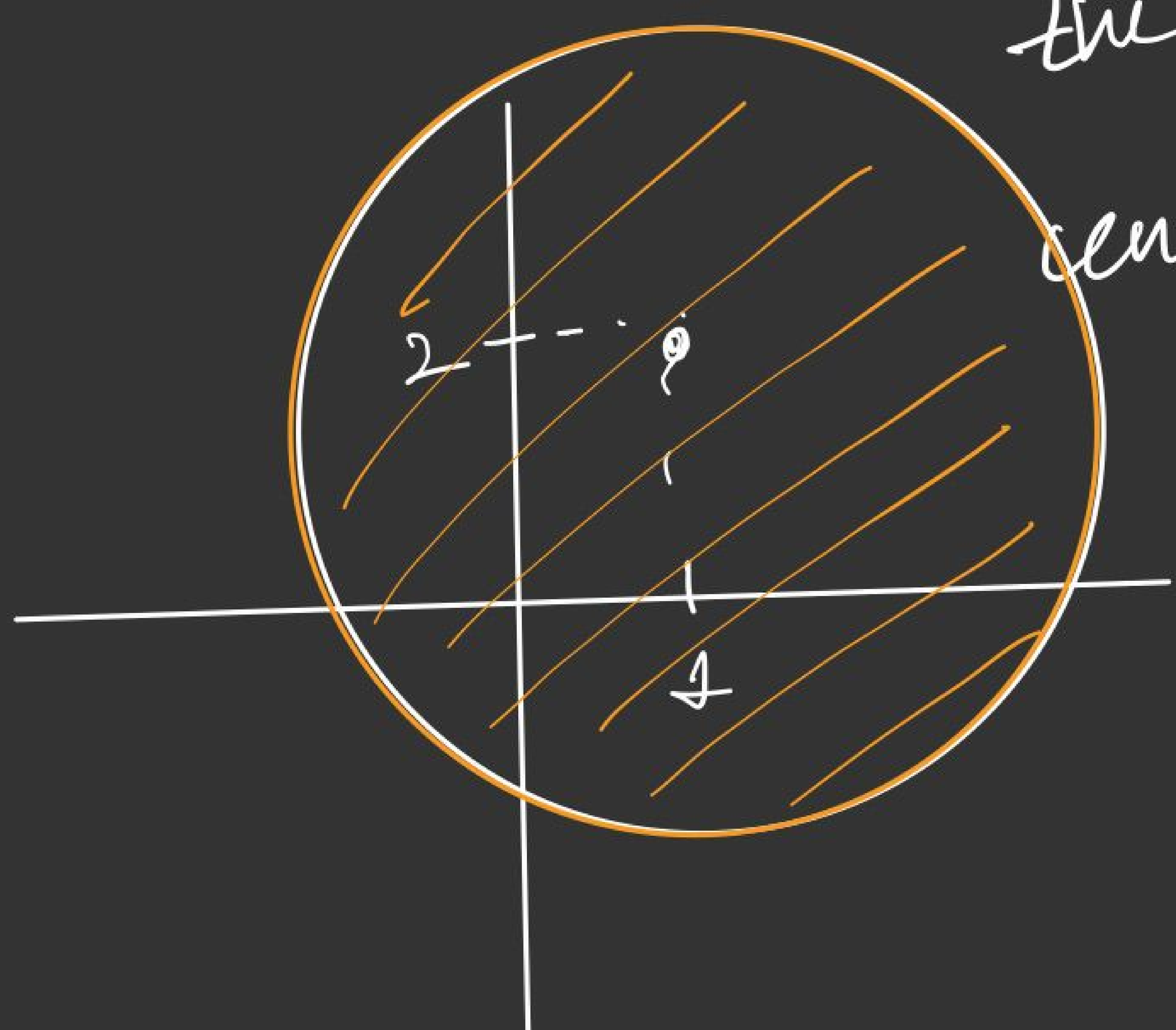
#5: Describe and sketch the regions defined by the followings:

(a) $x^2+y^2-2x-4y \leq 4.$

$$x^2-2x+\underline{1}+y^2-4y+\underline{4} \leq 4+5.$$

$$(x-1)^2+(y-2)^2 \leq 9 = 3^2.$$

the inside of the circle centered at (1,2) with radius 3.



(b) $x^2+y^2 \leq 4$ & $x^2+y^2 > 2y$.

$\downarrow$

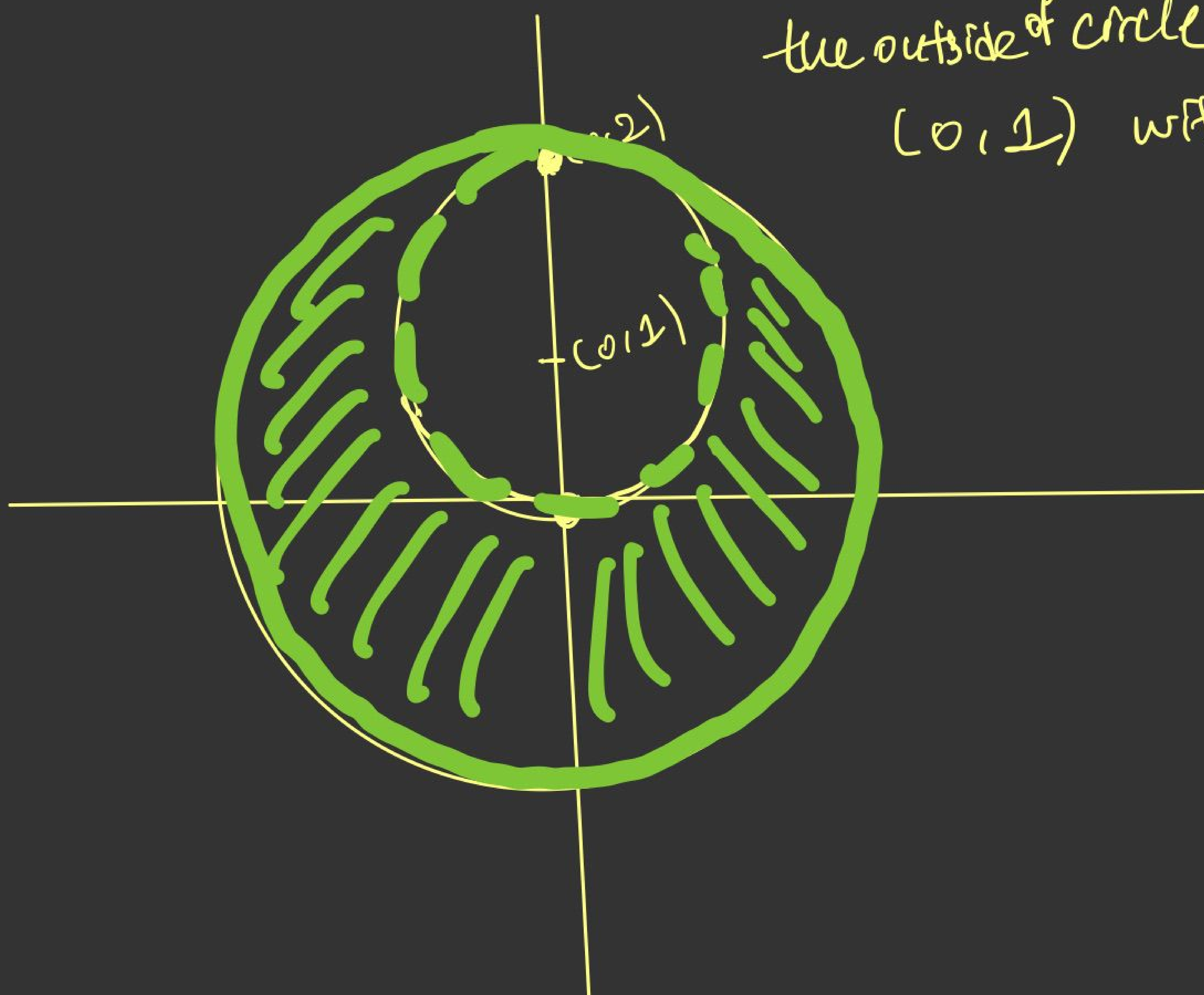
the inside of the circle centered at $(0,0)$ with radius 2.

$\underline{x^2+y^2-2y} > 0$

$x^2+y^2-2y+1 > 1$.

$x^2+(y-1)^2 > 1$.

the outside of circle centered at $(0,1)$ with radius 1.



(c) $x^2+y^2>2y$ & $y>1+x$.

$\downarrow$

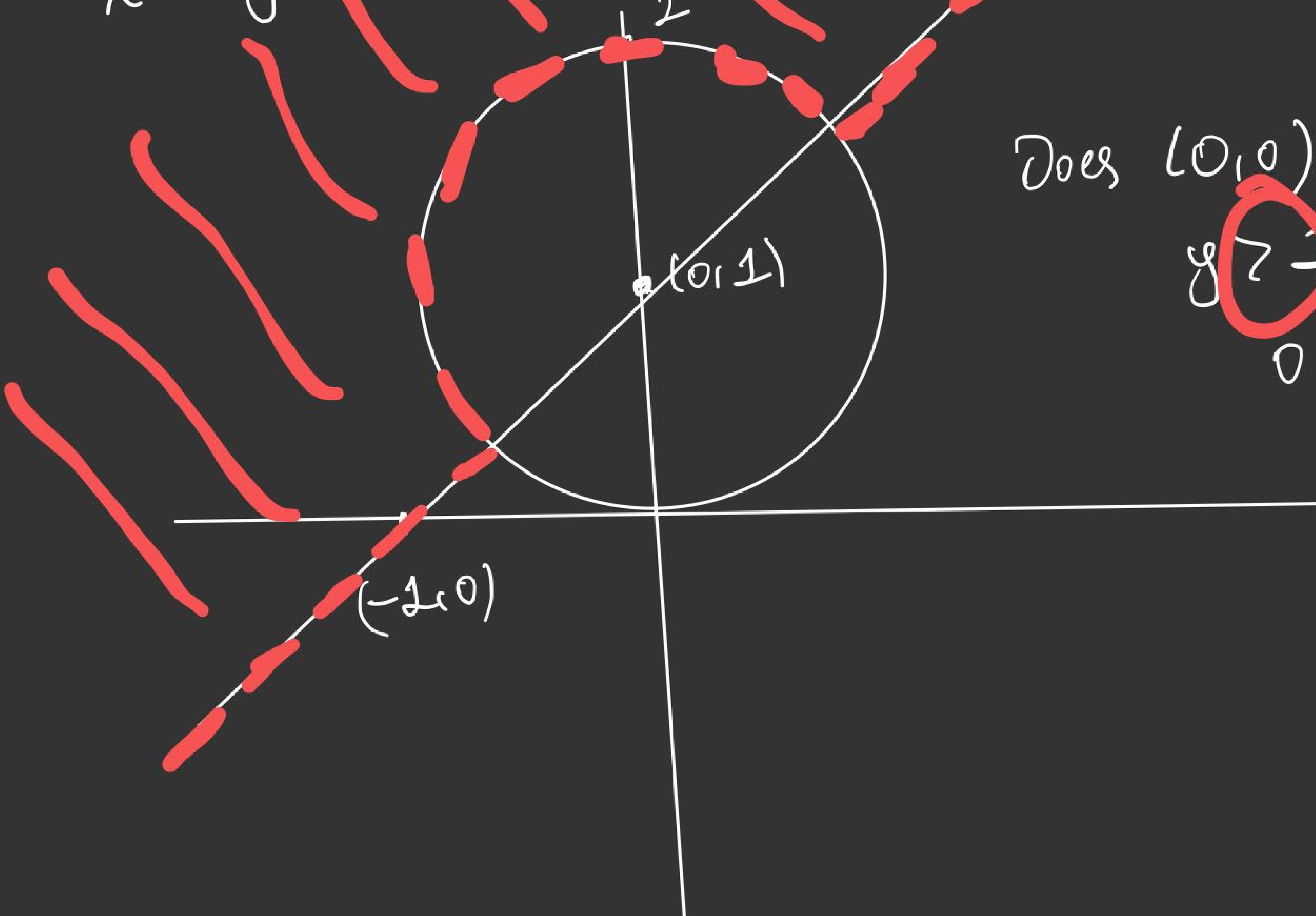
circle in part(b) with center $(0,1)$ & $r=1$.

$x^2+(y-1)^2>1 \rightarrow$ outer region.

$y=1+x$.

For $x=0 \Rightarrow y=1$. $(0,1)$

For $y=0 \Rightarrow x=-1$. $(-1,0)$



Does $(0,0)$ satisfy $y>1+x$?

$0>1+0$.

No!

#6: Find the points of intersection of pairs of curves

(a) $y = x^2 + 3$ & $y = 3x + 1$.

$x^2 + 3 = 3x + 1 \Rightarrow x^2 - 3x + 2 = 0$

$x \quad -1$

$x \quad -2$.

$(x-1)(x-2) = 0$.

$x = 1 \Rightarrow y = 4$

$x = 2 \Rightarrow y = 7$

$(1,4)$ & $(2,7)$ are the pts of intersection.

(b) $2x^2 + 2y^2 = 5$ (1) & $xy = 1$.

$xy = 1 \Rightarrow y = 1/x \Rightarrow$ put $y = 1/x$ in (1).

$(x \neq 0)$

$2x^2 + 2(1/x)^2 = 5 \Rightarrow 2x^2 + \frac{2}{x^2} = 5$.

$\Rightarrow \frac{2x^4 + 2}{x^2} = 5 \Rightarrow 2x^4 - 5x^2 + 2 = 0$

$2x^2 \quad -1$.

$x^2 \quad -2$

$(2x^2-1)(x^2-2)=0.$

$x^2 = 1/2$ & $x^2 = 2.$

$\Rightarrow$ $x = \mp 1/\sqrt{2}$ & $x = \mp\sqrt{2}.$

$\Downarrow$ $\Downarrow$

$y = \mp\sqrt{2}$ $y = \mp 1/\sqrt{2}.$

$\{ (1/\sqrt{2}, \sqrt{2}), (-1/\sqrt{2}, -\sqrt{2}).$

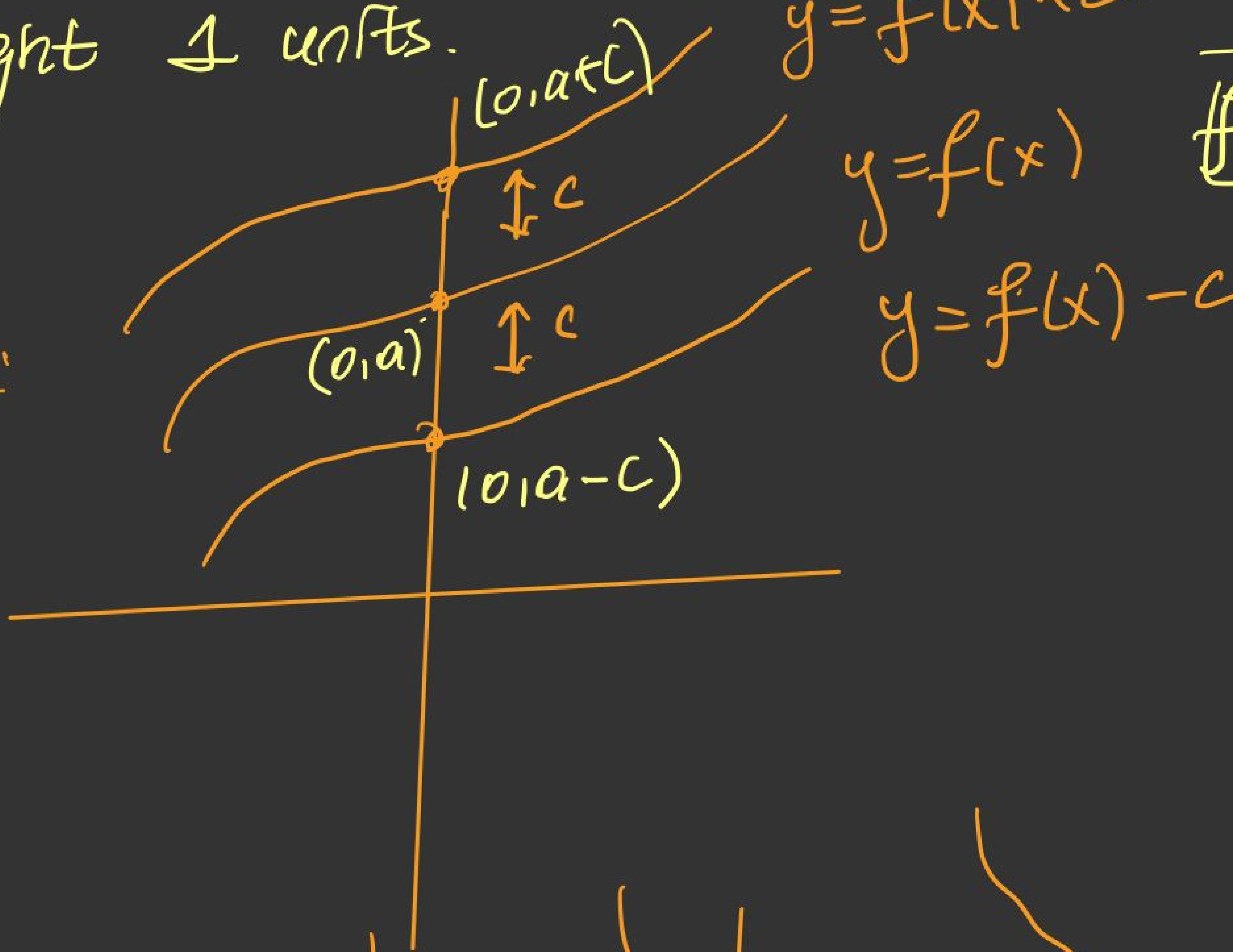
$(\sqrt{2}, 1/\sqrt{2})$ & $(-\sqrt{2}, -1/\sqrt{2}) \}$

pts of intersection.

#7: Write an equation of the graph obtained by shifting the graph of $y=\sqrt{x}$ by down 1 units & right 1 units.

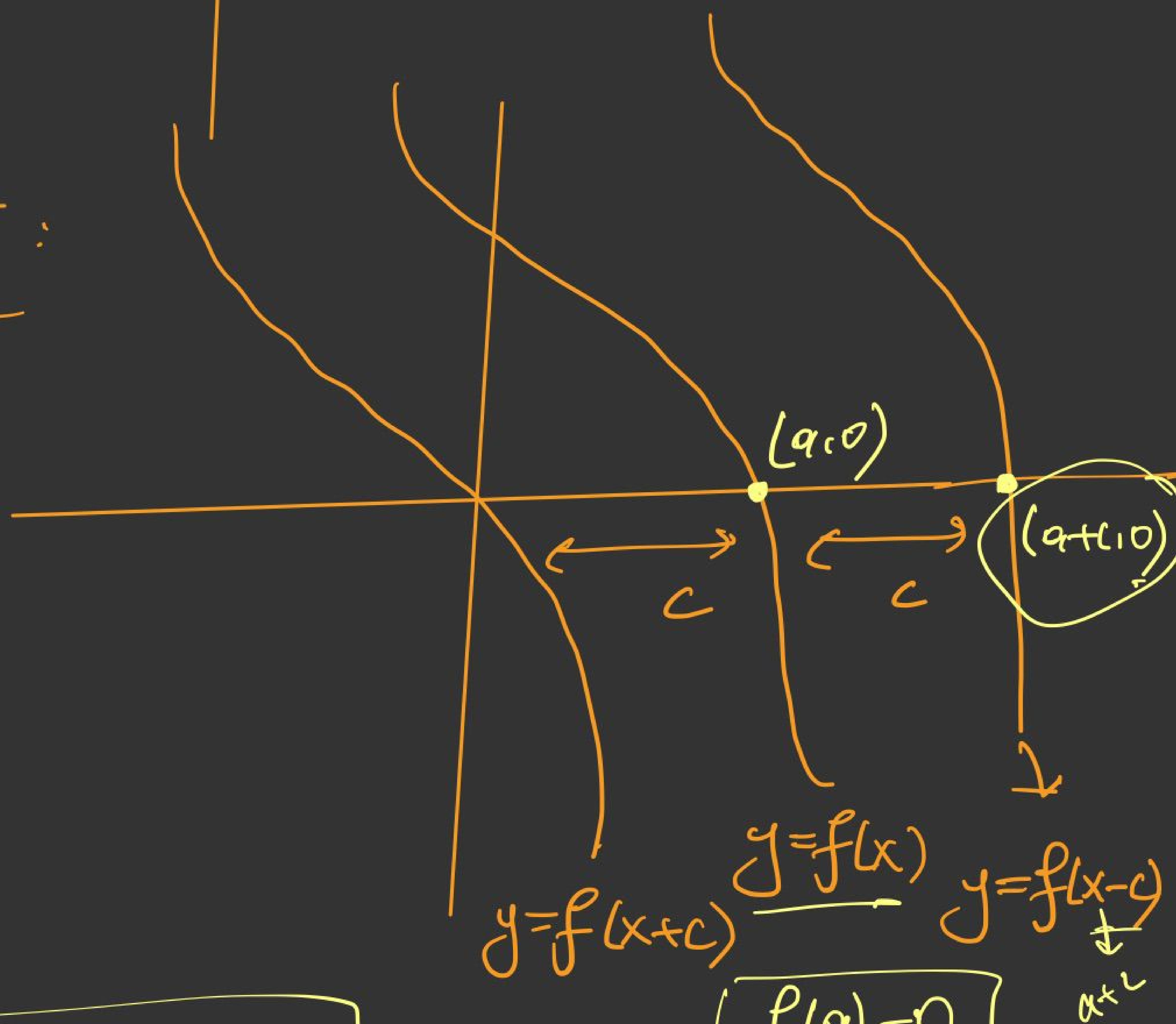
$y=f(x)$

Vertical shift:



$\boxed{f(0)=a.}$

Horizontal shift:



$\boxed{f(a)=0.}$

$$\boxed{y=\sqrt{x-1}-1.}$$

#8: Find the domain & range of each function and sketch their graphs:

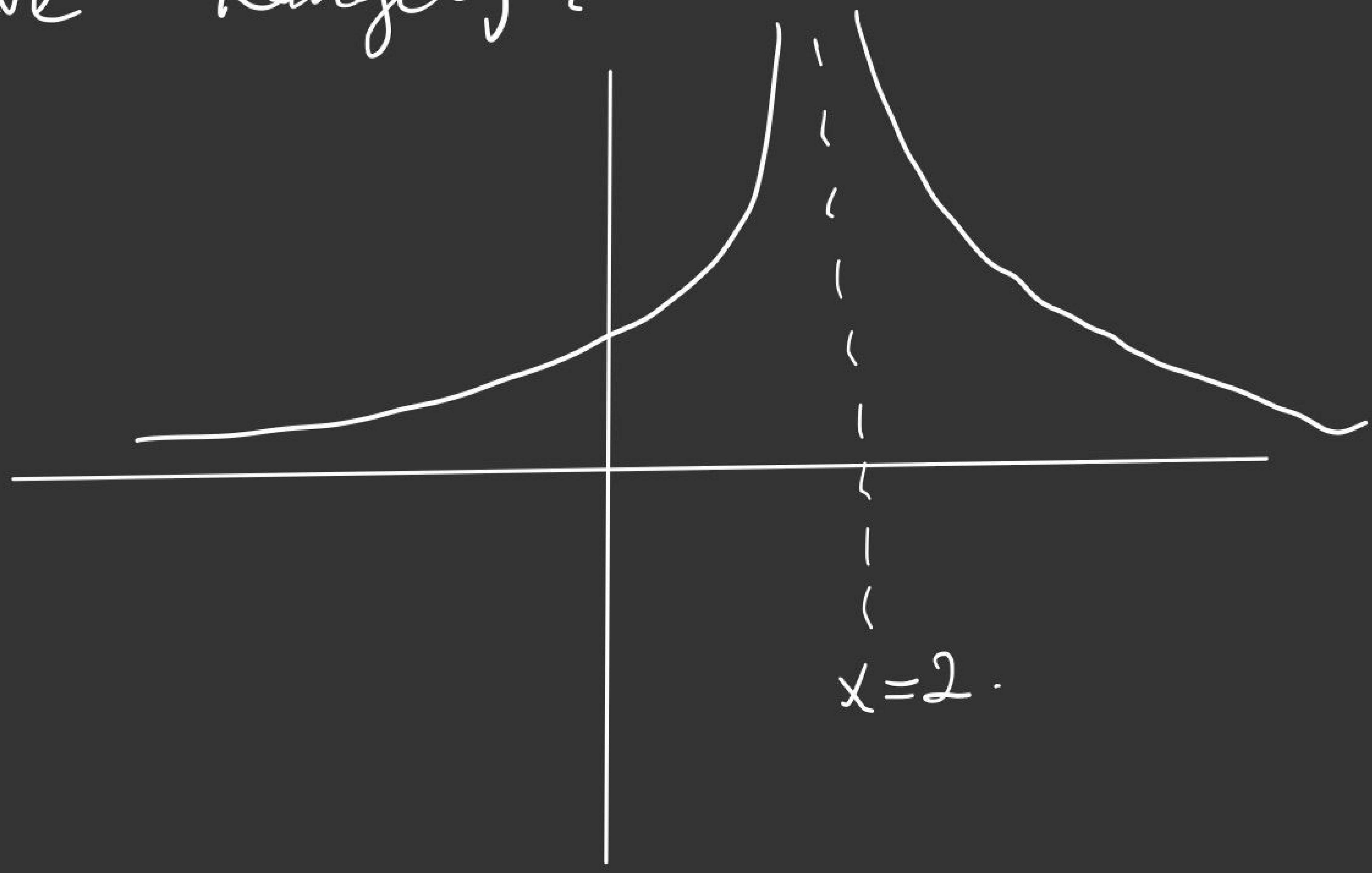
(a) $\dfrac{1}{|2-x|} = f(x)$.

Domain: Since f is defined everywhere on $\mathbb{R}$ except $x=2$, $\text{dom}(f) = \mathbb{R} \setminus \{2\}$.

Range: Since $\dfrac{1}{|2-x|} > 0$ for every $x \in \text{Dom} f$

and since $\dfrac{1}{|2-x|} \neq 0$ $\forall x \in \text{Dom} f$, we

have $\text{Range}(f) = (0, \infty)$.



(b) $y = 1 + \sin(x + \pi/4) = f(x)$

<u>Domain:</u> Since sine function is defined everywhere on $\mathbb{R}$, we will have $\text{Dom} f = \mathbb{R}$.

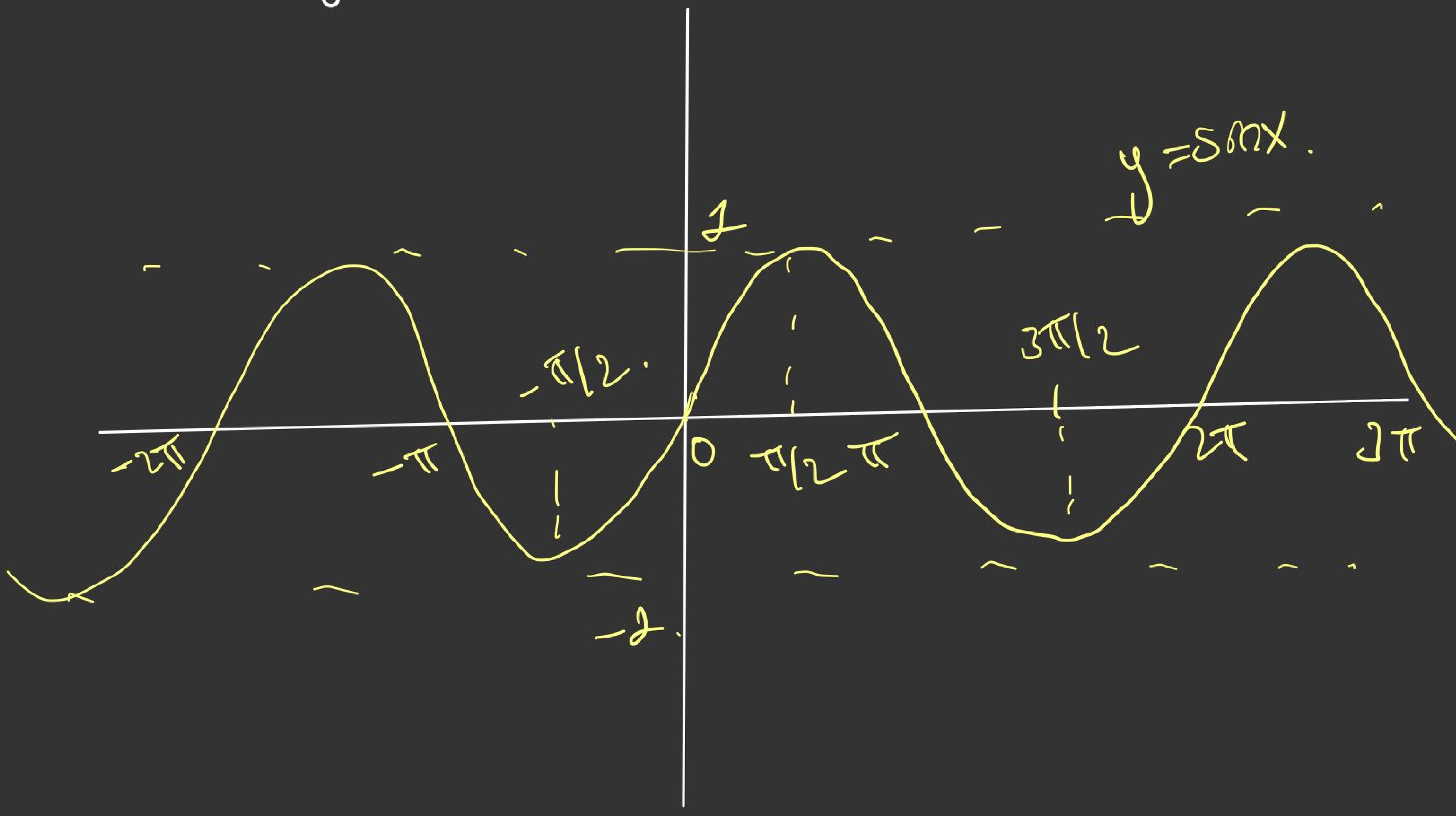
<u>Range:</u>

$$-1 \le \sin x \le 1 \quad \forall x \in \mathbb{R}.$$

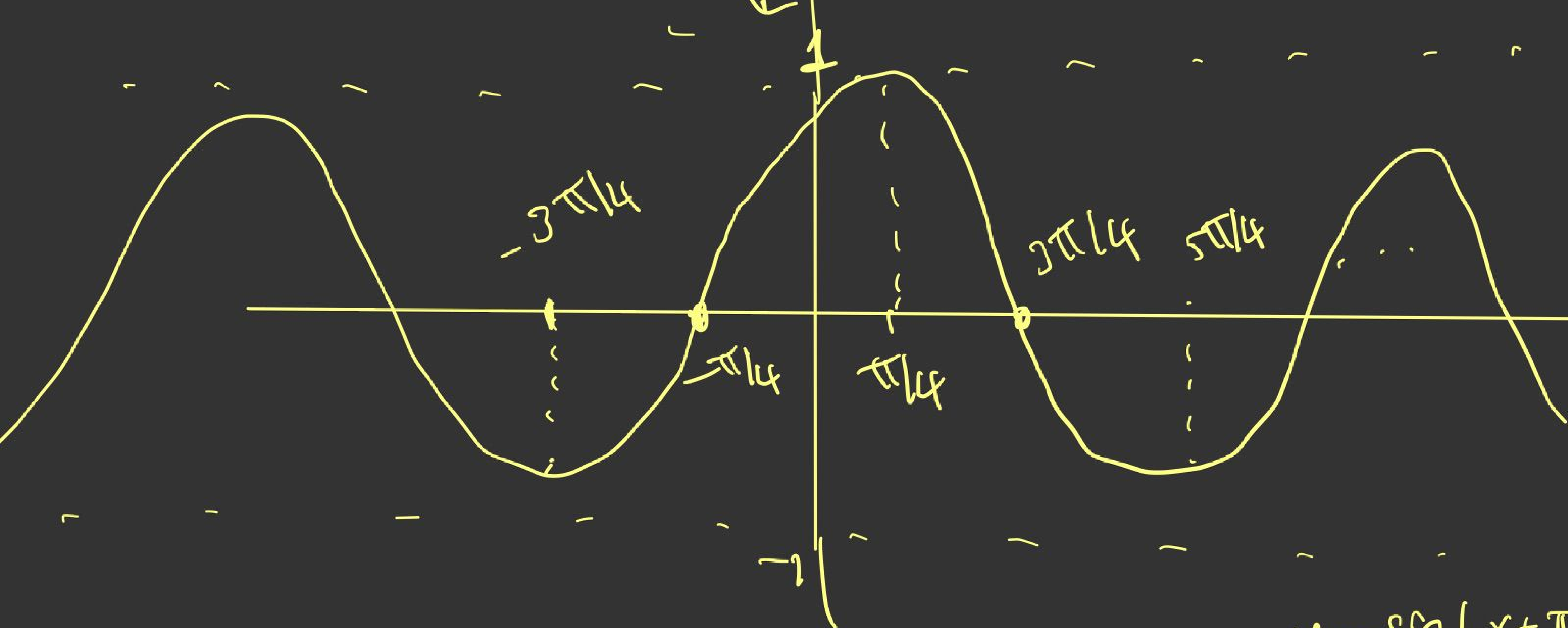
$$-1 \le \sin\left(x + \frac{\pi}{4}\right) \le 1 \quad \forall x \in \mathbb{R}.$$

$$0 \le \underbrace{1 + \sin\left(x + \frac{\pi}{4}\right)}_{f(x)} \le 2. \quad \forall x \in \mathbb{R}.$$

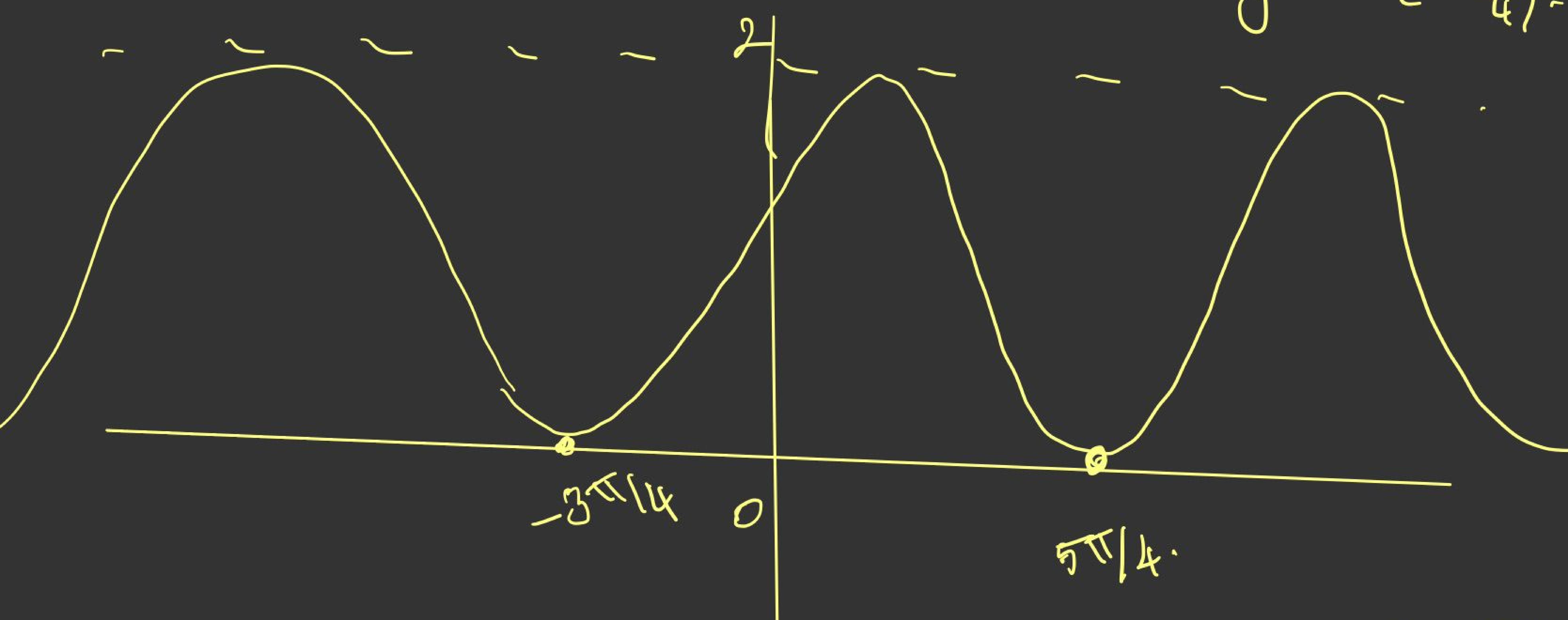
$\Rightarrow$ Range $(f) = [0, 2]$.



shift left by $\pi/4$.



$y = \sin\left(x + \frac{\pi}{4}\right)$.



$y = 1 + \sin(x + \pi/4)$

#9: Find $f \circ g$ and its domain where

$$f(x) = x + \frac{1}{x} \quad \& \quad g(x) = \frac{x-1}{x+3}.$$

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{x-1}{x+3}\right).$$

$$= \frac{x-1}{x+3} + \frac{x+3}{x-1} = \frac{x^2-2x+1+x^2+6x+9}{(x-1)(x+3)}$$

$(x-1)$ $(x+3)$

$$\boxed{= \frac{2x^2+4x+10}{(x-1)\cdot(x+3)}}$$

$$\text{Dom} f = \mathbb{R} - \{1, -3\}.$$