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Help Room Hours: Mon. 11.40 – 12.30

Tues. 14.40 – 15.30

REC - I

#1: Solve the following inequalities:

(a) $\frac{1}{2x+1} \geq 1-x$

$$\frac{1}{2x+1} + \frac{x-1}{\underset{(2x+1)}{1}} \geq 0 \Rightarrow \frac{1}{2x+1} + \frac{\overbrace{(x-1)(2x+1)}^{2x^2-x-1}}{2x+1} \geq 0.$$

$$\Rightarrow \frac{2x^2-x}{2x+1} \geq 0. \qquad f(x) = \frac{x(2x-1)}{2x+1} \geq 0.$$

$x = 0$

$x = 1/2$.

$x = -1/2$.

| $x$ | $-\infty$ | | $-1/2$ | | $0$ | | $1/2$ | | $+\infty$ |
|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | | $-$ | | $+$ | | $-$ | | $+$ | |

Solution set: $(-1/2, 0] \cup [1/2, \infty)$.

(b) $|x+3|-2>3x$

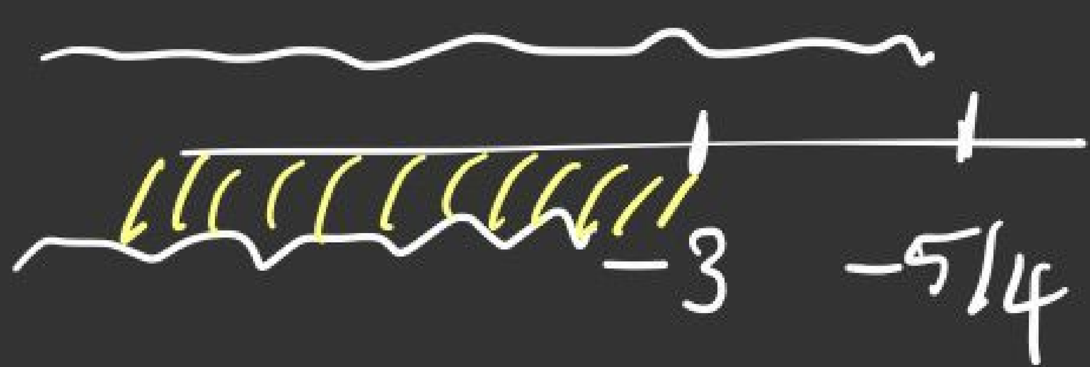
$$|x+3|>3x+2$$

when $x>-3$: $\quad x+3>3x+2 \Rightarrow 1>2x$

$\Rightarrow 1/2>x$. $\quad -3<x<1/2$. $\quad \boxed{(-3,\ 1/2)}$.

when $x<-3$: $\quad -x-3>3x+2 \Rightarrow -5>4x$

$\Rightarrow -5/4>x$.




$x<-3$. $\quad \boxed{(-\infty,-3)}$.

when $x=-3$: $\quad |\underbrace{-3+3}_{0}|-2>3\cdot(-3)$

$-2>-9$ ✓. $\quad \boxed{\{-3\}}$

Solution set: $(-\infty,-3)\cup\{-3\}\cup(-3,1/2)$.

$$=(-\infty,\ 1/2)$$

(c) $\dfrac{x^3-x^2+4}{x+3} \leq 1.$

$$\frac{x^3-x^2+4}{x+3} - 1 \leq 0. \Rightarrow \frac{x^3-x^2+4-(x+3)}{x+3} \leq 0.$$

$(x+3)$

$$\frac{x^3-x^2-x+1}{x+3} \leq 0. \Rightarrow \frac{x^2(x-1)-(x-1)}{x+3} \leq 0.$$

$$\Rightarrow \frac{(x-1)(x^2-1)}{x+3} \leq 0 \Rightarrow f(x) = \frac{(x-1)^2 \cdot (x+1)}{x+3} \leq 0.$$

$x = 1$ (double)

$x = -1$

$x = -3.$

| | $-\infty$ | | $-3$ | | $-1$ | | $1$ | | $+\infty$ |
|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | | $+$ | $\|$ | $-$ | $0$ | $+$ | $0$ | $+$ | |

Solution set: $(-3,-1] \cup \{1\}$.

#2: write an equation for the line through the points $(-1, 5)$ & $(0, 3)$.

$x_1 \quad y_1 \qquad x_2 \quad y_2$

$y = mx + b$
↓
slope.

$(x_1, y_1)$ & $(x_2, y_2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

$$m = \frac{3 - 5}{0 - (-1)} = -2.$$

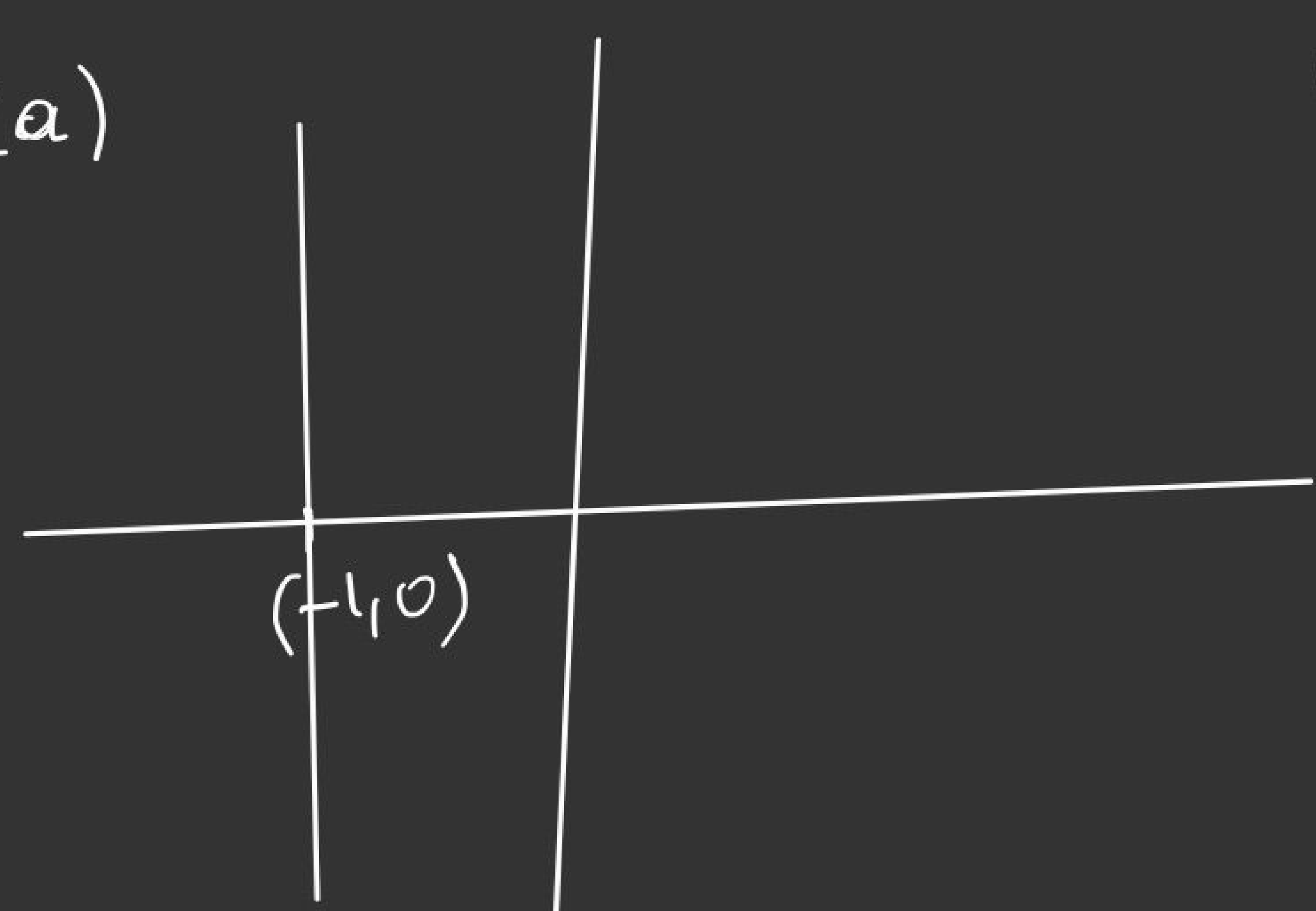
$y = -2x + b$

$(0, 3) \longrightarrow \quad 3 = -2 \cdot 0 + b \Rightarrow b = 3.$

$$\boxed{y = -2x + 3}.$$

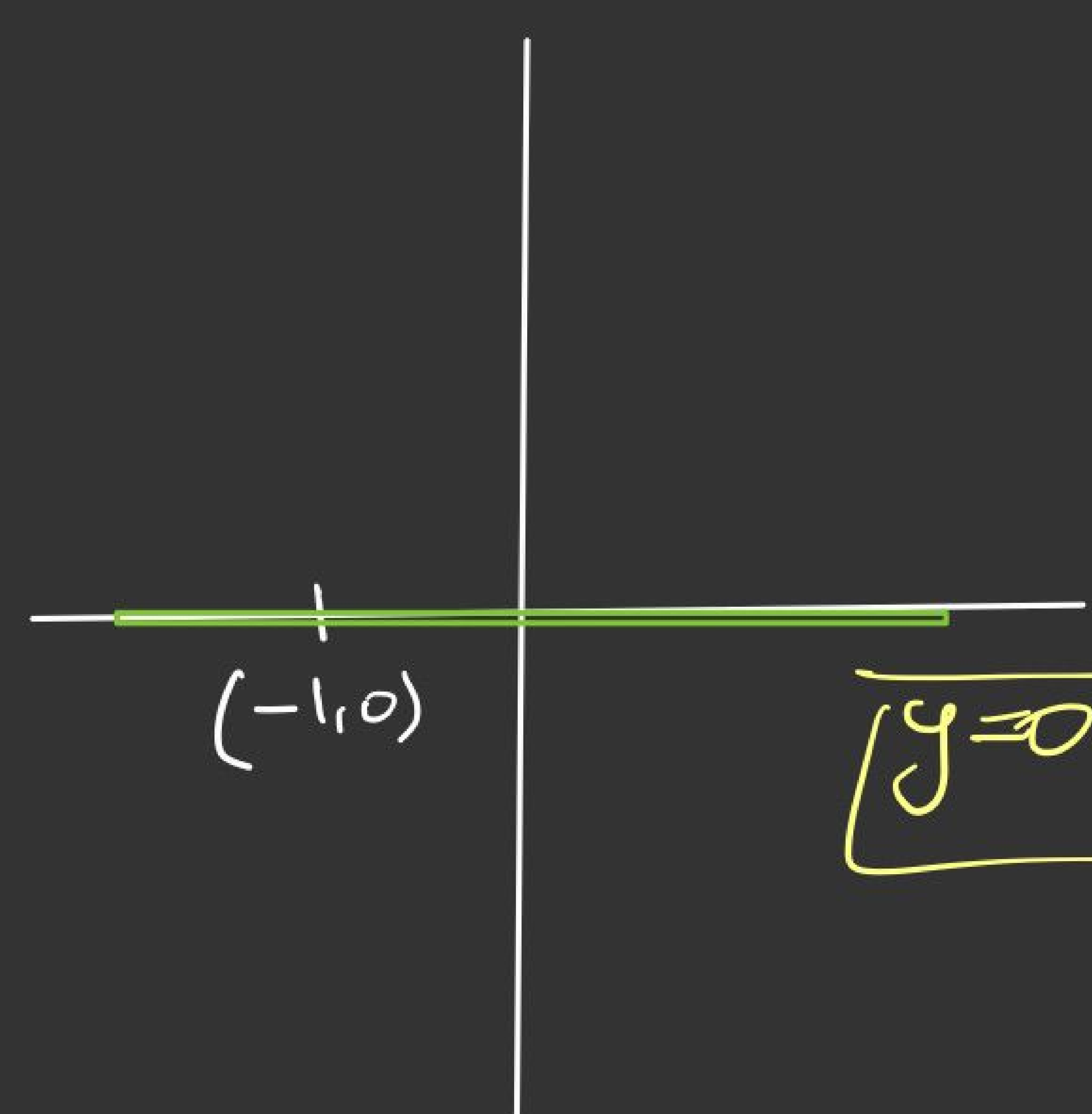
#3: Find the equation for (a) the vertical line & (b) the horizontal line through the point $(-1, 0)$.

(a)




$\boxed{x = -1}$.

(b)




$\boxed{y = 0}$

#4: Find the equation for the line through $P(-1,3)$ that is perpendicular to the line $y+x+2=0$. Find the x&y-intercepts of this line.

$$y = -x - 2 \qquad \boxed{m_1 = -1}$$

Let us call the slope of the desired line as $m_2$. Then since they are perpendicular, we must have $\underbrace{m_1}_{-1} \cdot m_2 = -1$.

$$\Rightarrow \boxed{m_2 = 1}$$

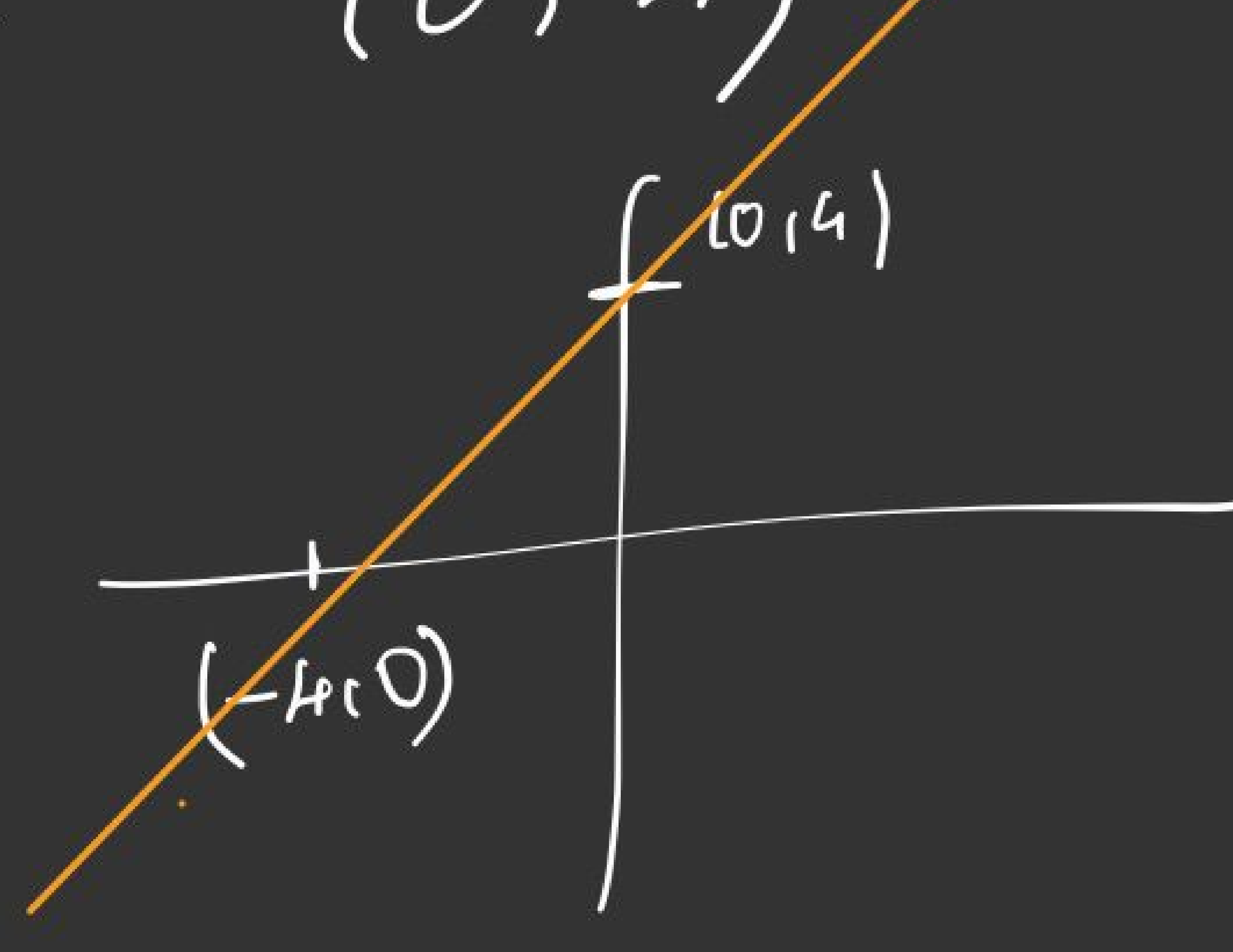
$$y = \underset{1}{\underset{\parallel}{m_2}} \cdot x + b = x + b \xrightarrow{(-1,3)} 3 = -1 + b$$

$$\boxed{b = 4}$$

$$\boxed{y = x + 4}$$

For x-intercept: $y = 0 \Rightarrow x = -4$ $\quad(-4, 0)$

For y-intercept: $x = 0 \Rightarrow y = 4$ $\quad(0, 4)$



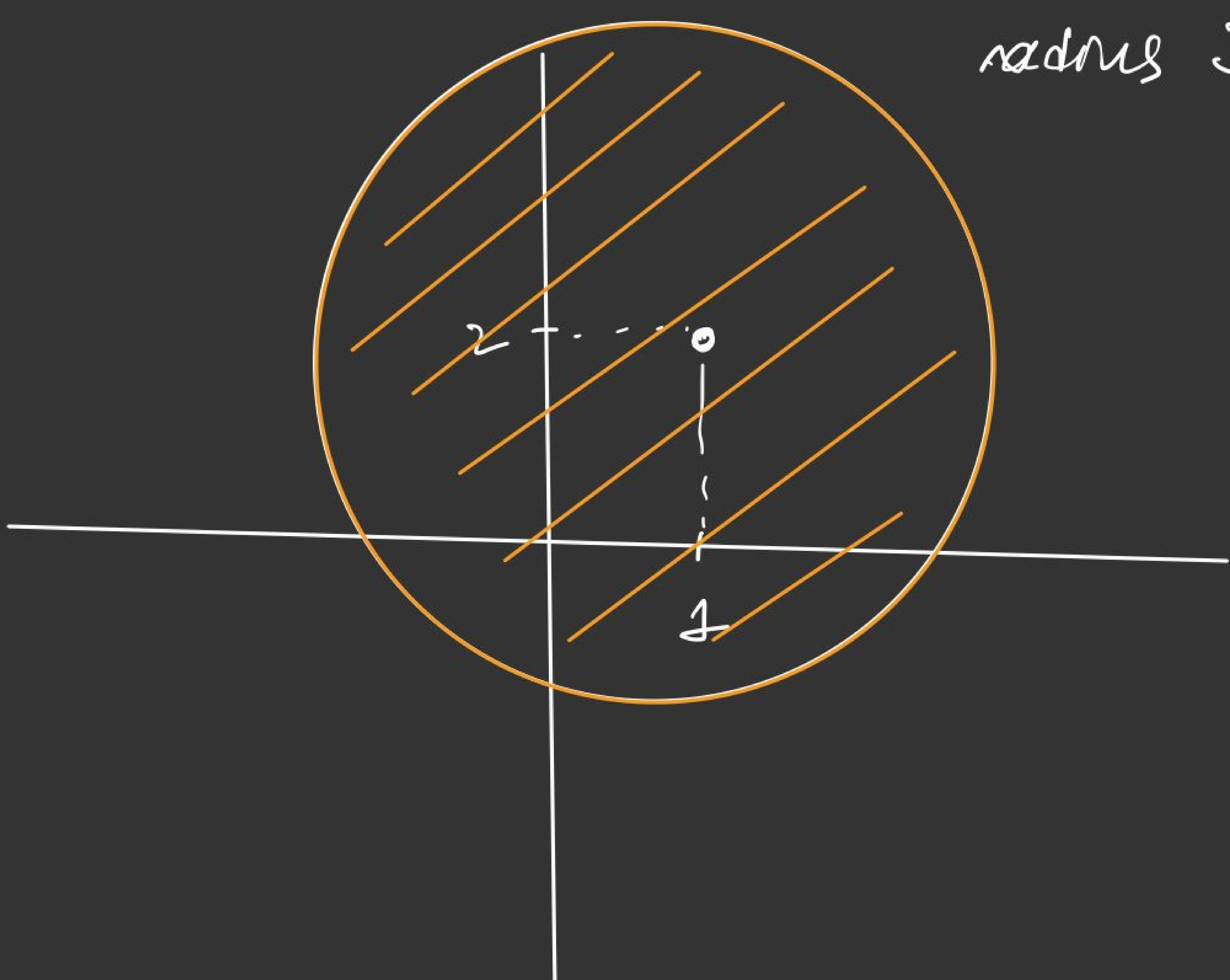
#5: Describe and sketch the regions defined by the followings:

(a) $x^2+y^2-2x-4y \le 4$.

$$x^2-2x+\underline{\underline{1}}+y^2-4y+\underline{\underline{4}} \le 4+5.$$

$$(x-1)^2+(y-2)^2 \le 9 = 3^2.$$

$(x-1)^2+(y-2)^2 = 3^2 \rightarrow$ the equation of the circle centered at $(1,2)$ with radius 3.



(b) $x^2+y^2 \le 4$ & $x^2+y^2 \ge 2y$.

$\downarrow$

the equ. of the circle centered at $(0,0)$ with $r=2$.

$\downarrow$

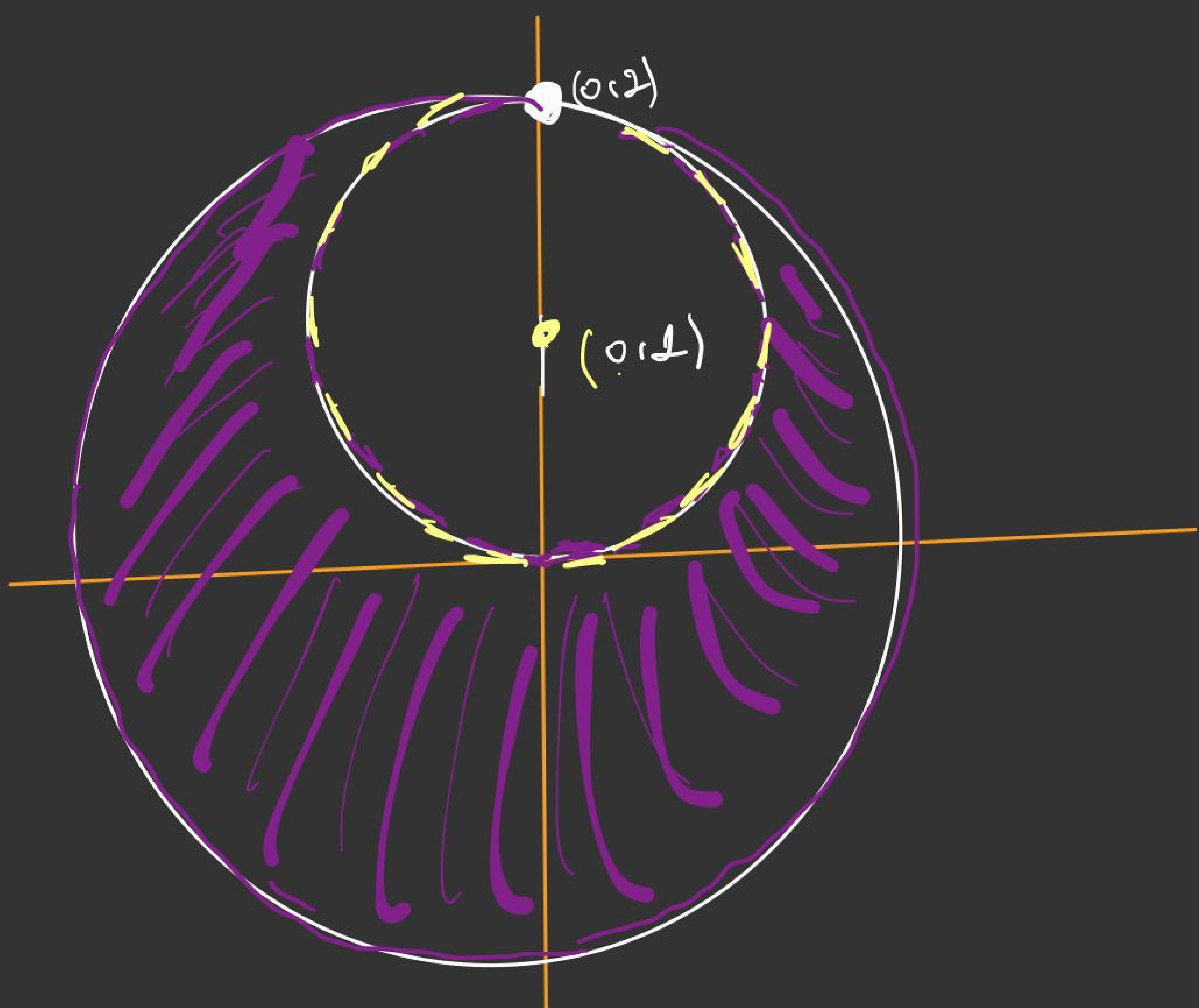
$x^2+y^2-2y \ge 0$.

$x^2+y^2-2y+1 \ge 1$

$x^2+(y-1)^2 \ge 1$.

$\downarrow$

the equ of a circle with center $(0,1)$ & $r=1$.




(c) Exercise: $x^2+y^2 \ge 2y$ & $y \ge 1+x$.

#6 : Find the points of intersection of pairs of curves.

$2x^2+2y^2=5$ ① & $x \cdot y = 1$.

$x \cdot y = 1 \Rightarrow y = 1/x \xrightarrow[\text{①}]{\text{put in}} 2x^2+2(1/x)^2=5$.

$(x \neq 0)$

$\Rightarrow 2x^2 + \dfrac{2}{x^2} = 5 \Rightarrow \dfrac{2x^4+2}{x^2} = 5$.

$\Rightarrow 2x^4+2 = 5x^2 \Rightarrow 2x^4 - 5x^2 + 2 = 0$.

$2x^2 \quad -1$

$x^2 \quad -2$

$\Rightarrow (2x^2-1)\cdot(x^2-2) = 0$.

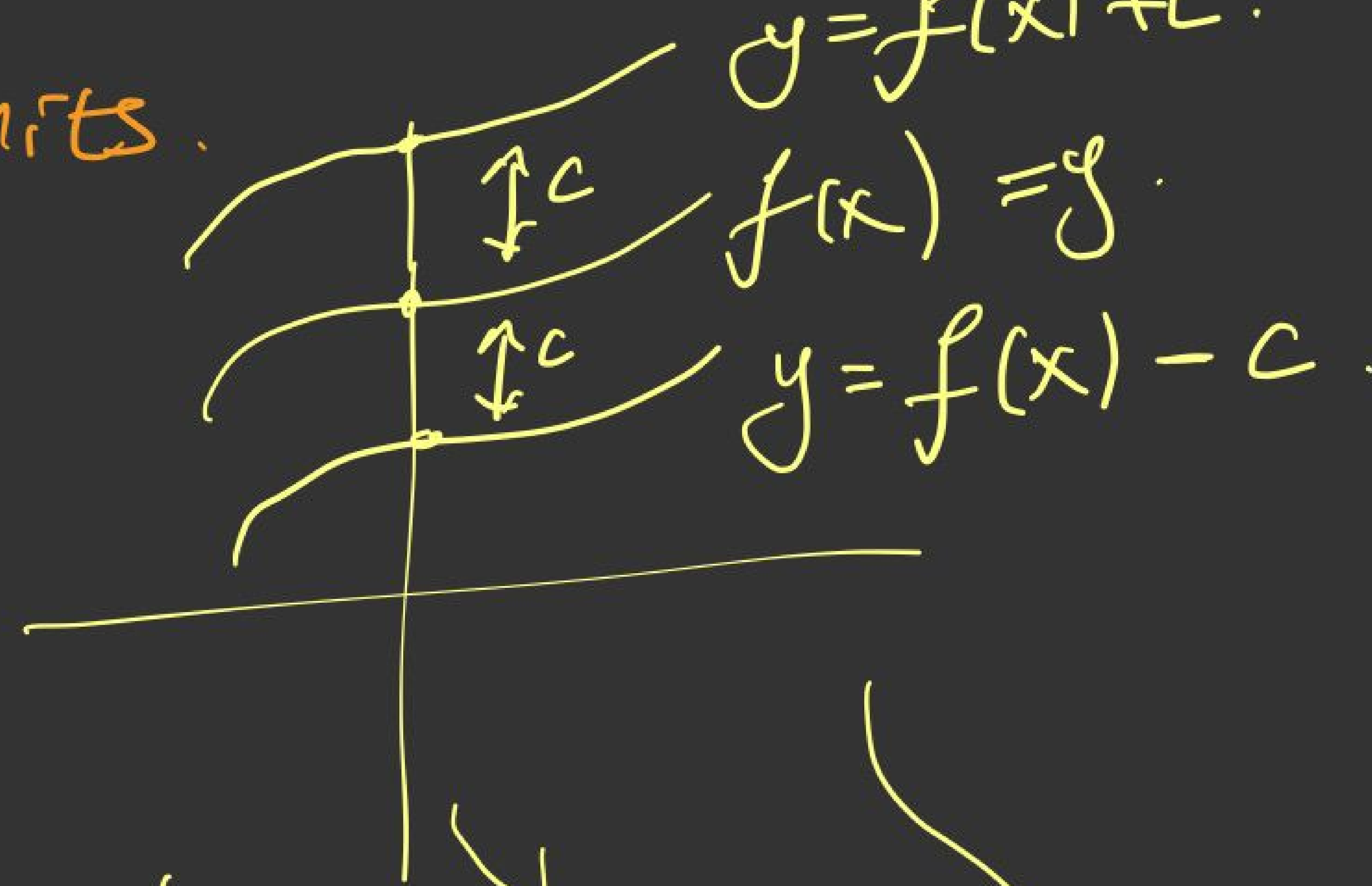
$x^2 = 1/2$ & $x^2 = 2 \Rightarrow x = \mp 1/\sqrt{2}$

& $x = \mp\sqrt{2}$.

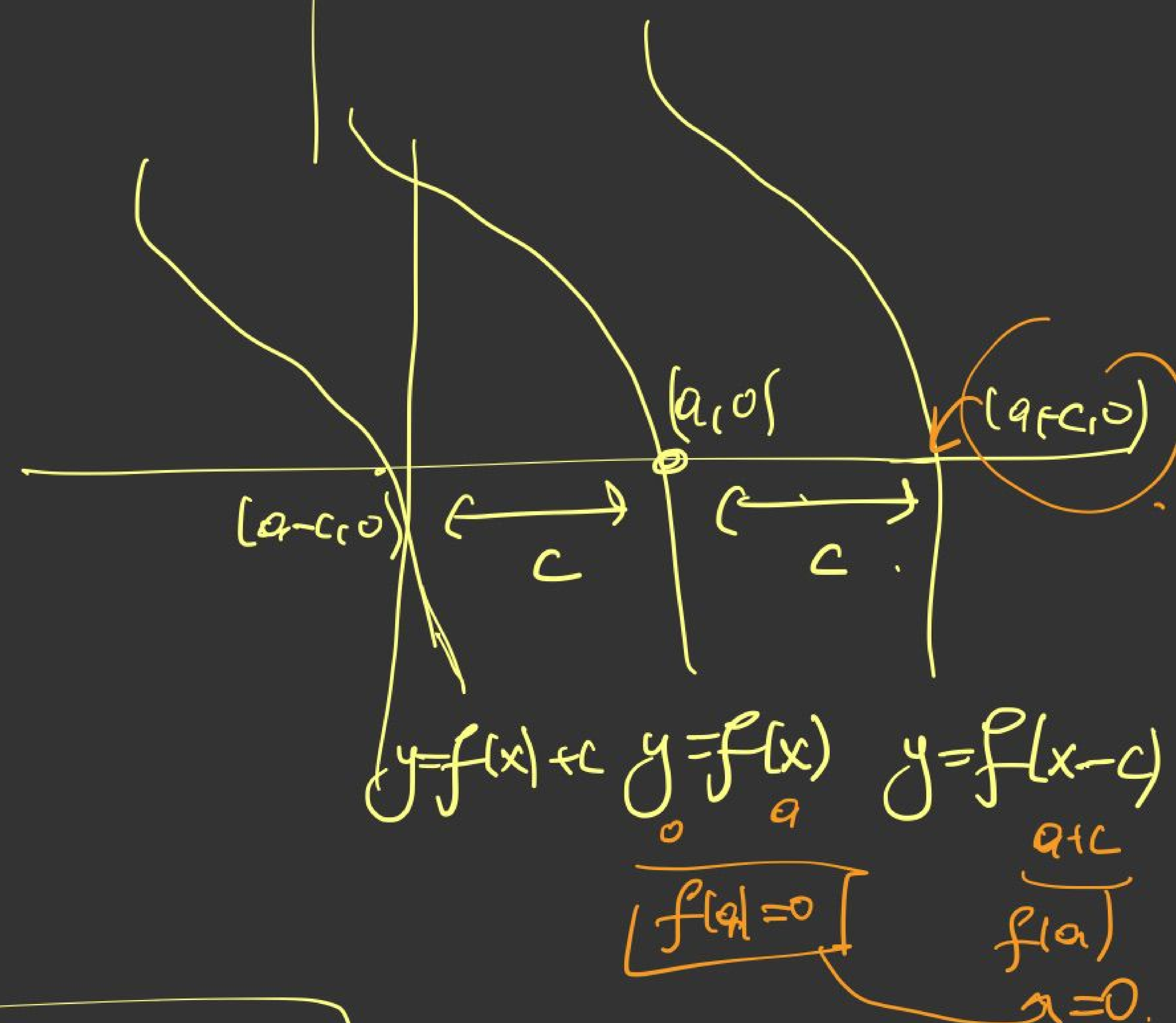
$\{(1/\sqrt{2}, \sqrt{2}), (-1/\sqrt{2}, -\sqrt{2}), (\sqrt{2}, 1/\sqrt{2}), (-\sqrt{2}, -1/\sqrt{2})\}$ points of intersection.

#7: Write an equation of the graph obtained by shifting the graph of $y=\sqrt{x}$ by down 1 units & right 1 units.

Vertical shift:




Horizontal shift:




$$\boxed{y=\sqrt{x-1}-1}$$

#8: Find the domain and range of each function and sketch their graphs:

(a) $\dfrac{1}{|2-x|} = f(x)$

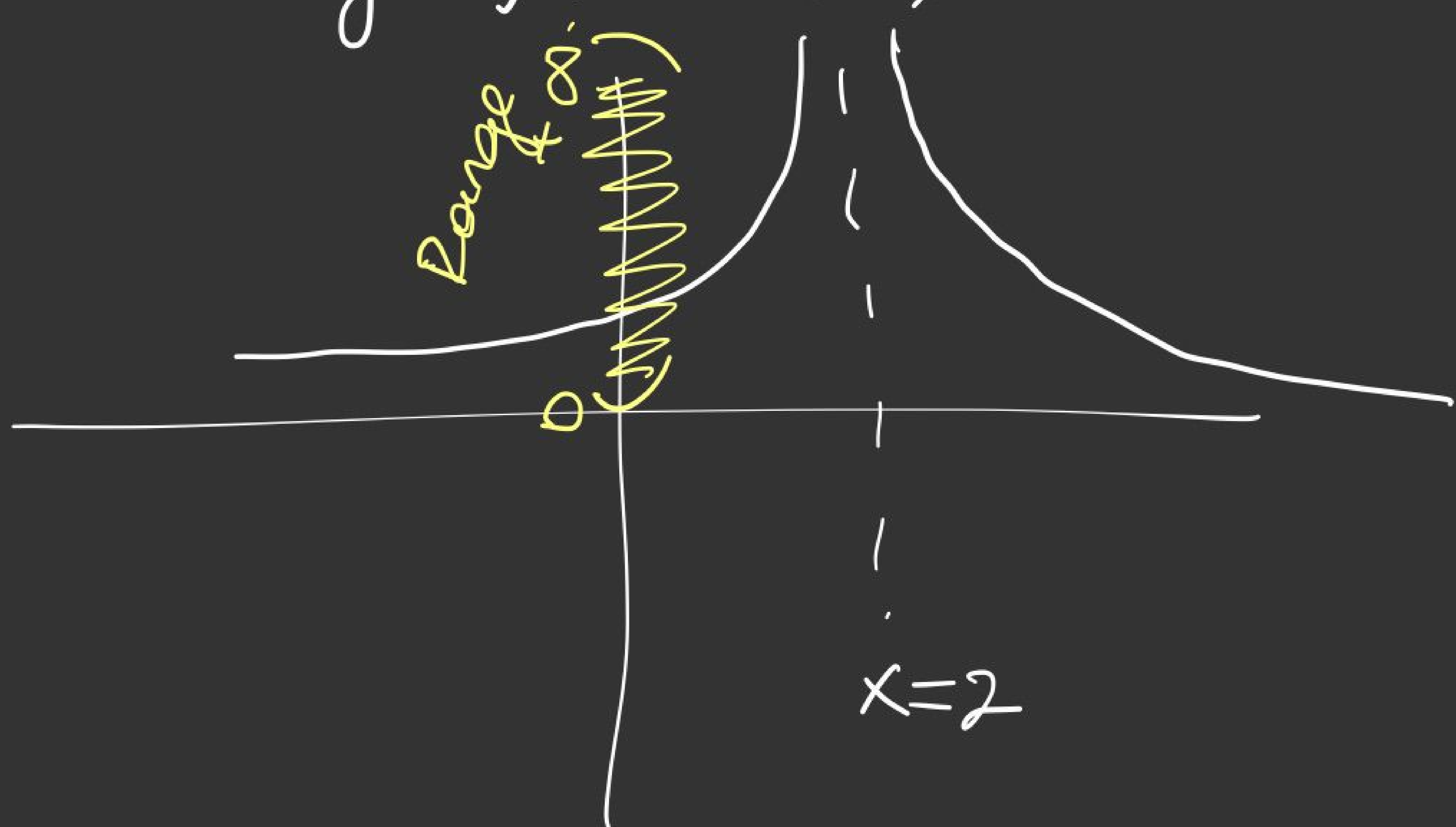
**Domain:** Since $f$ is not defined on $x=2$ but it is defined on everywhere else, we have

$$Dom f = \mathbb{R} - \{2\}.$$

**Range:** $|2-x| > 0 \ \forall x \in Dom f$ therefore,

$\dfrac{1}{|2-x|} > 0 \ \forall x \in Dom f$ and also, $\dfrac{1}{|2-x|} \neq 0 \ \forall x \in Dom f$

therefore $Range(f) = (0, \infty)$.



(b) $y = 1 + \sin\left(x + \frac{\pi}{4}\right) = f(x)$

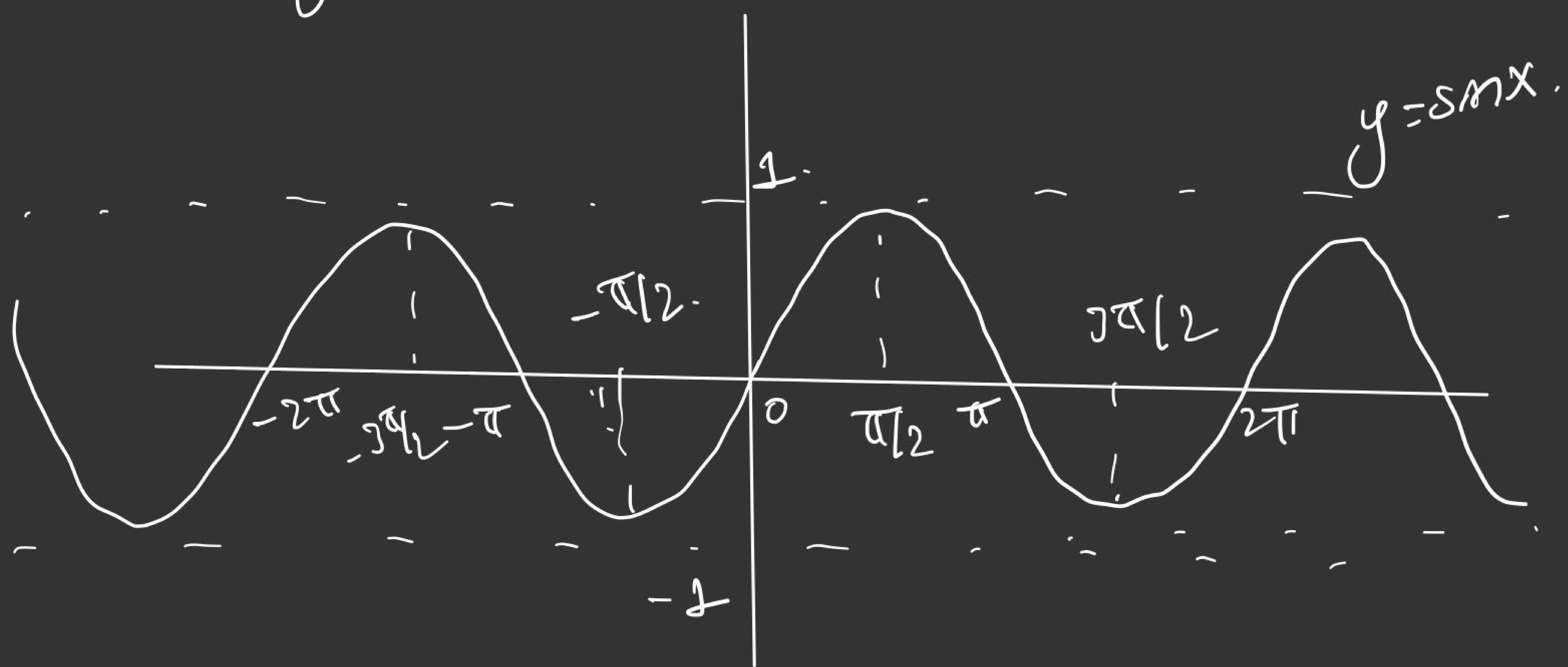
<u>Domain:</u> Since sine function is defined everywhere on $\mathbb{R}$, $f(x)$ is defined everywhere on $\mathbb{R}$ also. Therefore, $\text{Dom} f = \mathbb{R}$.

<u>Range:</u> $-1 \leq \sin x \leq 1 \quad \forall x \in \mathbb{R}$.

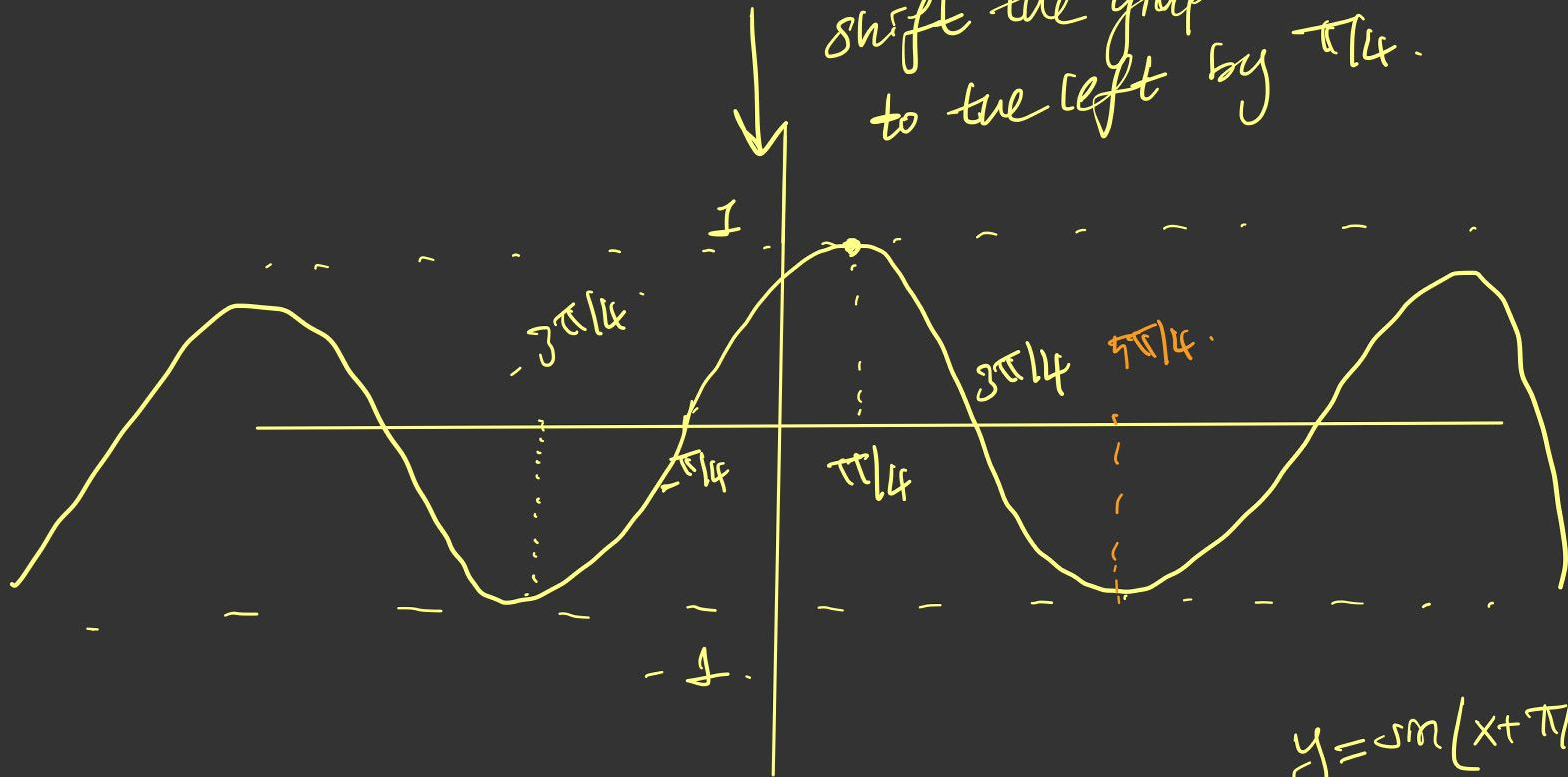
$$-1 \leq \sin\left(x + \frac{\pi}{4}\right) \leq 1 \quad \forall x \in \mathbb{R}.$$

$$0 \leq \underbrace{1 + \sin\left(x + \frac{\pi}{4}\right)}_{f(x)} \leq 2 \quad \forall x \in \mathbb{R}.$$

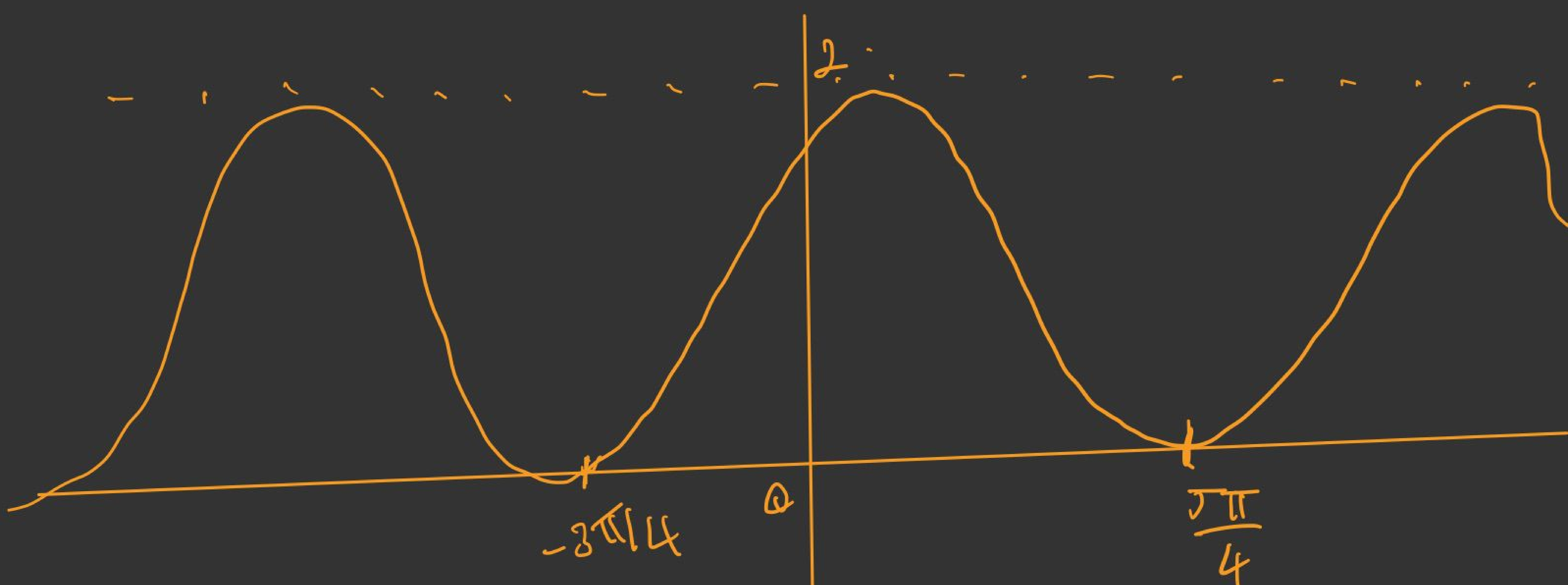
$\text{Range}(f) = [0, 2]$.



shift the graph to the left by $\pi/4$.




$y = \sin(x + \pi/4)$




$$y = \underbrace{1 + \sin(x + \pi/4)}_{= f(x)}$$

#9: (a) Find $f \circ g$ and its domain where

$f(x) = x + \frac{1}{x}$ and $g(x) = \frac{x-1}{x+3}$.

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{x-1}{x+3}\right)$$

$$= \frac{x-1}{x+3} + \frac{x+3}{x-1} = \frac{x^2 - 2x + 1 + x^2 + 6x + 9}{(x-1)(x+3)}$$

$$= \frac{2x^2 + 4x + 10}{(x-1)(x+3)}.$$

Domain: $\mathrm{Dom} f = \mathbb{R} \setminus \{-3, 1\}$.

(b) Given $F(x) = \sin^2(x-5)$, find functions $f, g, h$ such that $F = f \circ g \circ h$.

$f(x) = x^2$. $g(x) = \sin x$ & $h(x) = x - 5$.

$$(f \circ g \circ h)(x) = (f \circ g)(h(x)) = (f \circ g)(x-5) = f(g(x-5))$$

$$= f(\sin(x-5)) = \sin^2(x-5) = \underline{\underline{F(x)}}.$$