

Math 466 Exercises for Week 12

May 11, 2025

NOTE: Solve exercise 1 before exercise 2, this will be very helpful!

1. Consider the rotation matrices $A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ and $B = \frac{1}{9} \begin{bmatrix} -7 & 4 & 4 \\ 4 & -1 & 8 \\ 4 & 8 & -1 \end{bmatrix}$.

For each of these, find the rotation axis and angle of rotation.

2. Let us find a general way of finding the rotation angle of a rotation matrix:

Recall that the trace $tr(A)$ of a matrix A is the sum of its diagonal elements.

(a) Show that $tr(AB) = tr(BA)$ and $tr(P^{-1}AP) = tr(A)$.

(b) Given a rotation matrix $A \in \mathbb{R}^{3 \times 3}$ with rotation angle α , show that there is a matrix P such that

$$P^{-1}AP = \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(c) Find a formula for α and compare with your answer to Ex. 1.

3. Show that $SO(3)$ is not an abelian group.

I may add more exercises during the week!!