Math 466 Exercises for Week 1

February 21, 2025

1. Let (X, d) be a metric space. If $f : X \to X$ preserves distances (i.e., $d(x, y) = d(f(x), f(y))$ for all $x, y \in X$) show that f must be injective.
2. Find a metric space (X, d) and a function $f : X \to X$ which preserves distances but is not surjective.

   (A fact for the interested students: Every function $f : \mathbb{R}^n \to \mathbb{R}^n$ which preserves distances is bijective.)
3. Let $f, g \in Isom(\mathbb{R})$ and $x, y \in \mathbb{R}$ with $x \neq y$. If $f(x) = g(x)$ and $f(y) = g(y)$, show that $f = g$. (i.e., an isometry of $\mathbb{R}$ is uniquely determined by its action on two distinct points.)
4. Find two reflections $a, b \in D_n(D_\infty)$ which generate $D_n(D_\infty)$.

   (Recall that if G is a group, its *center* is $Z(G) = \{g \in G \mid gh = hg \;\forall\, h \in G\} \leq G$
5. Show that the center of D_n is $\{e\}$ if n is odd and $\{e, r^{n/2}\}$ if n is even.
6. Show that the center of D_∞ is $\{e\}$.