

Introduction to Atiyah-Singer Index Theorem

Batuhan Kaynak Acar¹

¹) Department of Mathematics
Middle East Technical University, 06800 Ankara, Turkey
kaynak.acar@metu.edu.tr

The eigenvalue problem of an (elliptic) operator on a manifold certainly depends on the topology of the manifold it is defined on. However, is it possible to study the topology of a manifold by studying the eigenvalue problem of differential operators? de Rham cohomology and harmonic analysis are two examples of this((Hou & Hou, 1999)). Atiyah-Singer(AS) Index Theorem takes this idea further, relating the analytical index of an elliptic differential operator with a topological invariant given by an integral of some characteristic classes. In this talk, we will start by reviewing some of the well-known characteristic classes. Then proceed by introducing elliptic and Fredholm operators and the analytic index of a Fredholm operator. Then we will define an elliptic complex over a vector bundle and state the AS Index theorem. Finally, we will derive the Gauss-Bonnet and Riemann-Roch theorems using the index theorem.

References

Hou, B.-Y., & Hou, B.-Y. (1999). *Differential geometry for physicists* (Vol. 6).